M-22. Polumjer kružnice kojoj su asimptote hiperbole $4x^2 - 9y^2 = 12$ tangente, a središte joj je u žarištu te hiperbole, iznosi

A. $\frac{3}{2}$ B. $\frac{2}{\sqrt{3}}$ C. $\frac{3}{\sqrt{2}}$ D. $\sqrt{3}$ E. $\sqrt{2}$

$$4x^2 - 9y^2 = 12 \quad /:12$$

$$\frac{4x^2}{12} - \frac{9y^2}{12} = 1$$

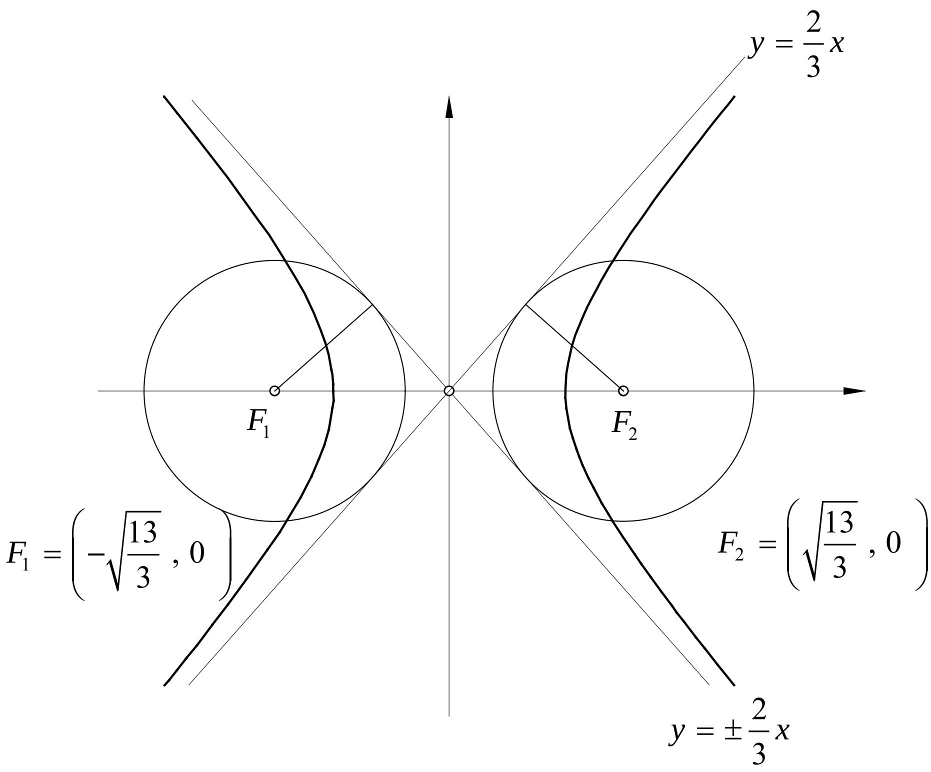
$$\frac{x^2}{3} - \frac{3y^2}{4} = 1$$

$$\frac{x^2}{3} - \frac{y^2}{\frac{4}{3}} = 1$$

$$\Downarrow \quad \Downarrow$$

$$a^2 = 3 \quad b^2 = \frac{4}{3}$$

$$a = \sqrt{3} \quad b = \frac{2}{\sqrt{3}}$$



$$e^2 = a^2 + b^2$$

$$e^2 = 3 + \frac{4}{3}$$

$$e^2 = \frac{13}{3} \quad /\sqrt{\ }$$

$$e = \pm\sqrt{\frac{13}{3}}$$

$$F = \left(\pm\sqrt{\frac{13}{3}}, 0\right)$$

jednadžba asimptota glasi:

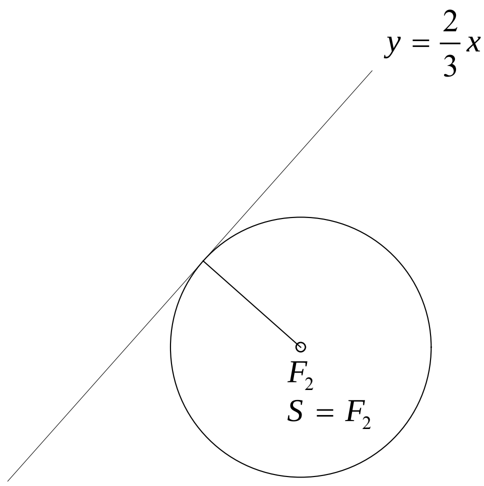
$$y = \pm\frac{b}{a}x$$

$$y = \pm\frac{\frac{2}{\sqrt{3}}}{\sqrt{3}}x = \pm\frac{\frac{2}{\sqrt{3}}}{\frac{\sqrt{3}}{1}}x = \pm\frac{2\cdot 1}{\sqrt{3}\cdot\sqrt{3}}x = \pm\frac{2}{\sqrt{3^2}}x = \pm\frac{2}{3}x$$

$$y = \pm\frac{2}{3}x$$

Imamo dva fokusa i dvije asimptote svejedno je koju uzmemo za dalji račun

Ja ću uzeti F_2 i $y = \frac{2}{3}x$



$F_2 = \left(\sqrt{\frac{13}{3}}\ ,\ 0 \right)$

$k = \frac{2}{3},\ \ l = 0$ $\qquad\qquad$ $S = (\ p\ ,\ q\ ) \Rightarrow p = \sqrt{\frac{13}{3}}\ ,\ \ q = 0$

$\uparrow$ $\qquad\qquad\qquad\qquad$ $\Uparrow$

Ako je pravac $y = \frac{2}{3}x$ tangenta kružnice sa središtem u $F_2 = \left(\sqrt{\frac{13}{3}}\ ,\ 0 \right)$

Tada mora biti zadovoljen uvjet dodira tangente i kružnice:

$$(-kp + q - l)^2 = r^2 \left(1 + k^2\right)$$

$$\left(-\frac{2}{3} \cdot \sqrt{\frac{13}{3}} + 0 - 0 \right)^2 = r^2 \cdot \left(1 + \left(\frac{2}{3}\right)^2 \right)$$

$$\left(-\frac{2}{3} \cdot \sqrt{\frac{13}{3}} \right)^2 = r^2 \cdot \left(1 + \frac{4}{9} \right)$$

$$\left(-\frac{2}{3} \right)^2 \cdot \sqrt{\left(\frac{13}{3}\right)}^2 = r^2 \cdot \frac{9+4}{9}$$

$$\frac{4}{9} \cdot \frac{13}{3} = r^2 \cdot \frac{13}{9} \qquad / \cdot \frac{9}{13}$$

$$\frac{4 \cdot 13 \cdot 9}{9 \cdot 3 \cdot 13} = r^2$$

$$\frac{4}{3} = r^2$$

$$r = \sqrt{\frac{4}{3}}$$

$$r = \frac{2}{\sqrt{3}}$$