

POTPUNO RIJEŠENI ZADACI



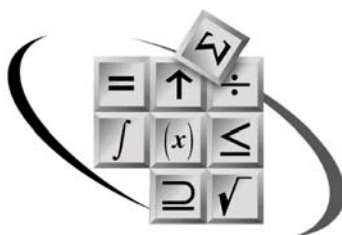
PRIRUČNIK ZA SAMOSTALNU PRIPREMU PRIJEMNOG ISPITA NA TEHNIČKE FAKULTETE

1997./98.g.

PRAVOKUTNI TROKUT		MATEMATIČKE FORMULE ZA TREĆI RAZRED SREDNJE ŠKOLE		PARKIJE VIŠESTRUKOG IČKUPITA																																																																																																																																																																				
<p> $\sin \alpha = \frac{a}{c}$ $\cos \alpha = \frac{b}{c}$ $\operatorname{tg} \alpha = \frac{a}{b}$ $\operatorname{ctg} \alpha = \frac{b}{a}$ </p>	<p> $\sin \alpha = \frac{a}{c}$ $\cos \alpha = \frac{b}{c}$ $\operatorname{tg} \alpha = \frac{a}{b}$ $\operatorname{ctg} \alpha = \frac{b}{a}$ </p>	<p>Adicijske formule</p> <p> $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$ $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$ $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$ $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$ $\operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \operatorname{tg} \beta}$ $\operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \operatorname{tg} \beta}$ $\operatorname{ctg}(\alpha + \beta) = \frac{\operatorname{ctg} \alpha \operatorname{ctg} \beta - 1}{\operatorname{ctg} \alpha + \operatorname{ctg} \beta}$ $\operatorname{ctg}(\alpha - \beta) = \frac{\operatorname{ctg} \alpha \operatorname{ctg} \beta + 1}{\operatorname{ctg} \alpha - \operatorname{ctg} \beta}$ </p>	<p>Transformacija zbroja u umnožak</p> <p> $\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$ $\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$ $\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$ $\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$ $\operatorname{tg} \alpha + \operatorname{tg} \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta}$ $\operatorname{tg} \alpha - \operatorname{tg} \beta = \frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta}$ $\operatorname{ctg} \alpha + \operatorname{ctg} \beta = \frac{\sin(\alpha + \beta)}{\sin \alpha \sin \beta}$ $\operatorname{ctg} \alpha - \operatorname{ctg} \beta = \frac{-\sin(\alpha - \beta)}{\sin \alpha \sin \beta}$ </p>	<p>Transformacija umnoška u zbroj</p> <p> $\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$ $\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$ $\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$ $\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$ </p>	<p>Parnost, neparnost i periodičnost</p> <p> $\sin(-x) = -\sin x$ $\cos(-x) = \cos x$ $\operatorname{tg}(-x) = -\operatorname{tg} x$ $\operatorname{ctg}(-x) = -\operatorname{ctg} x$ </p> <p> $\sin(x + 2k\pi) = \sin x$ $\cos(x + 2k\pi) = \cos x$ $\operatorname{tg}(x + k\pi) = \operatorname{tg} x$ $\operatorname{ctg}(x + k\pi) = \operatorname{ctg} x$ </p>																																																																																																																																																																			
<p>Pokrajnje kružnice:</p> <p> $r = \frac{c}{2} \frac{a}{2 \sin \alpha} = \frac{b}{2 \cos \alpha}$ $\rho = c \sin \frac{\alpha}{2} (\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2})$ </p>	<p>Inverzne formule:</p> <p> $a = c \sin \alpha = b \operatorname{tg} \alpha = c \cos \beta$ $a = b \operatorname{ctg} \beta = \frac{b}{\operatorname{ctg} \alpha}$ $b = c \cos \alpha = c \sin \beta = a \operatorname{tg} \beta$ $b = a \operatorname{ctg} \alpha = \frac{a}{\operatorname{tg} \beta}$ $c = \frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{a}{\cos \beta} = \frac{b}{\cos \alpha}$ </p>	<p>Transformacija zbroja u umnožak</p> <p> $\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$ $\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$ $\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$ $\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$ $\operatorname{tg} \alpha + \operatorname{tg} \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta}$ $\operatorname{tg} \alpha - \operatorname{tg} \beta = \frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta}$ $\operatorname{ctg} \alpha + \operatorname{ctg} \beta = \frac{\sin(\alpha + \beta)}{\sin \alpha \sin \beta}$ $\operatorname{ctg} \alpha - \operatorname{ctg} \beta = \frac{-\sin(\alpha - \beta)}{\sin \alpha \sin \beta}$ </p>	<p>Parkije višestrukog ičkupita</p> <p> $\sin 2\alpha = 2 \sin \alpha \cos \alpha$ $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$ $\operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$ $\operatorname{ctg} 2\alpha = \frac{\operatorname{ctg} \alpha - 1}{2 \operatorname{ctg} \alpha}$ $\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$ $\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$ $\operatorname{tg} 3\alpha = \frac{3 \operatorname{tg} \alpha - \operatorname{tg}^3 \alpha}{1 - 3 \operatorname{tg}^2 \alpha}$ $\operatorname{ctg} 3\alpha = \frac{\operatorname{ctg} \alpha - 3 \operatorname{ctg}^3 \alpha}{3 \operatorname{ctg}^2 \alpha - 1}$ $\sin 4\alpha = 2 \sin 2\alpha \cos 2\alpha$ $\cos 4\alpha = \cos^2 2\alpha - \sin^2 2\alpha$ $\operatorname{tg} \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$ $\operatorname{ctg} \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{1 - \cos \alpha}}$ $\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$ $\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$ </p>																																																																																																																																																																					
<p>Stupajevci u radijane</p> <p> $x^\circ = x \frac{\pi}{180}$ </p> <p>Radijani u stupajevce</p> <p> $x(\text{rad}) = \frac{180^\circ}{\pi} x$ </p>		<p>Čitanje grafa</p> <p> $y = a \sin(bx + c)$ </p> <ol style="list-style-type: none"> Nacrtamo graf $y = \sin x$ Amplitudni sinusoidi povećamo a puta Period smanjimo b puta Sinusoidi $y = a \sin bx$ pomaknemo c duž x-osi na $(-\frac{c}{b})$ 		<p>Uklopana suprotnosti</p> <p> $\sin 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 + \operatorname{tg}^2 \alpha}$ $\cos 2\alpha = \frac{1 - \operatorname{tg}^2 \alpha}{1 + \operatorname{tg}^2 \alpha}$ $\operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$ </p> <p>ili</p> <p>Ako je $\operatorname{tg} \frac{\alpha}{2} = t$</p> <p>Tada je $\sin \alpha = \frac{2t}{1+t^2}$, $\cos \alpha = \frac{1-t^2}{1+t^2}$, $\operatorname{tg} \alpha = \frac{2t}{1-t^2}$</p>																																																																																																																																																																				
<p>Osnovne relacije</p> <p> $\sin^2 \alpha + \cos^2 \alpha = 1$ $1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha}$ $1 + \operatorname{ctg}^2 \alpha = \frac{1}{\sin^2 \alpha}$ </p> <p> $\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$ $\operatorname{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha}$ $\operatorname{tg} \alpha \operatorname{ctg} \alpha = 1$ </p>	<p>Veza između funkcija istoglana</p> <table border="1"> <thead> <tr> <th>Parkija</th> <th>$\sin \alpha$</th> <th>$\cos \alpha$</th> <th>$\operatorname{tg} \alpha$</th> <th>$\operatorname{ctg} \alpha$</th> </tr> </thead> <tbody> <tr> <td>$\sin \alpha$</td> <td>$\sin \alpha$</td> <td>$\pm \sqrt{1 - \cos^2 \alpha}$</td> <td>$\frac{\operatorname{tg} \alpha}{1 \pm \operatorname{tg} \alpha}$</td> <td>$\frac{1}{\pm \sqrt{1 + \operatorname{ctg}^2 \alpha}}$</td> </tr> <tr> <td>$\cos \alpha$</td> <td>$\pm \sqrt{1 - \sin^2 \alpha}$</td> <td>$\cos \alpha$</td> <td>$\frac{1}{\pm \sqrt{1 + \operatorname{tg}^2 \alpha}}$</td> <td>$\frac{\operatorname{ctg} \alpha}{1 \pm \operatorname{ctg} \alpha}$</td> </tr> <tr> <td>$\operatorname{tg} \alpha$</td> <td>$\frac{\sin \alpha}{\pm \sqrt{1 - 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<p>Preduzi po kvadrantima</p> <table border="1"> <thead> <tr> <th></th> <th>I.</th> <th>II.</th> <th>III.</th> <th>IV.</th> </tr> </thead> <tbody> <tr> <td>$\sin \varphi$</td> <td>+</td> <td>+</td> <td>-</td> <td>-</td> </tr> <tr> <td>$\cos \varphi$</td> <td>+</td> <td>-</td> <td>-</td> <td>+</td> </tr> <tr> <td>$\operatorname{tg} \varphi$</td> <td>+</td> <td>-</td> <td>+</td> <td>-</td> </tr> <tr> <td>$\operatorname{ctg} \varphi$</td> <td>+</td> <td>-</td> <td>+</td> <td>-</td> </tr> </tbody> </table>		I.	II.	III.	IV.	$\sin \varphi$	+	+	-	-	$\cos \varphi$	+	-	-	+	$\operatorname{tg} \varphi$	+	-	+	-	$\operatorname{ctg} \varphi$	+	-	+	-																																																																																																																																															
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25. Kamen mase 4 kg bačen je vertikalno prema dolje s visine od 120 m početnom brzinom $v_0 = 10 \text{ m/s}$. Kolika je energija potrebna za svladavanje otpora zraka, ako kamen udari o zemlju brzinom $v = 20 \text{ m/s}$?

A. 3.909 kJ B. 3.709 kJ C. 5.309 kJ D. 5.709 kJ E. 4.109 kJ

$$m = 4 \text{ kg}$$

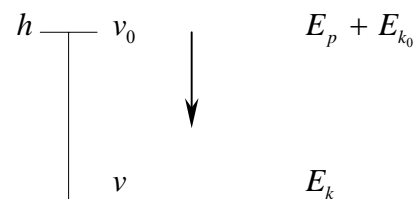
$$h = 120 \text{ m}$$

$$v_0 = 10 \text{ m/s}$$

$$v = 20 \text{ m/s}$$

$$W = E = ?$$

verikalan hitac prema dolje:



$$E_p = m \cdot g \cdot h = 4 \text{ kg} \cdot 9,81 \text{ m/s}^2 \cdot 120 \text{ m} = 4708,8 \text{ J} = 4709 \text{ J}$$

$$E_{k_0} = \frac{mv_0^2}{2} = \frac{4 \text{ kg} \cdot (10 \text{ m/s})^2}{2} = 2 \text{ kg} \cdot 100 \text{ m}^2/\text{s}^2 = 200 \text{ J}$$

$$E_k = \frac{mv^2}{2} = \frac{4 \text{ kg} \cdot (20 \text{ m/s})^2}{2} = 2 \text{ kg} \cdot 400 \text{ m}^2/\text{s}^2 = 800 \text{ J}$$

$$W = E_p + E_{k_0} - E_k = 4709 \text{ J} + 200 \text{ J} - 800 \text{ J} = 4109 \text{ J} = 4,109 \text{ J}$$

26. Nakon koliko će se vremena aktivnost 1 g izotopa radija ${}_{88}\text{Ra}^{226}$ smanjiti za 20%, ako je vrijeme poluraspada tog izotopa 1622 godine?

- A. 1298 god B. 522 god C. 811 god D. 406 god E. 324 god

$$m({}_{88}\text{Ra}^{226}) = 1 \text{ g}$$

$$T_{1/2} = 1622 \text{ god}$$

$$N_1 = 20\%$$

$$t = ?$$

N_0 - početni broj čestica

N_0 - 100%

N - broj čestica koji se nije raspao

$$N = N_0 - N_1 = 100\% - 20\% = 80\%$$

$$N = N_0 \cdot 2^{-\frac{t}{T_{1/2}}} \quad / \log$$

$$\log N = \log N_0 + \log 2^{-\frac{t}{T_{1/2}}}$$

$$\log N = \log N_0 - \frac{t}{T_{1/2}} \log 2$$

$$\log 80 = \log 100 - \frac{t}{1,622} \cdot 0,3010$$

$$1,903 = 2 - \frac{0,3010}{1,622} t$$

$$\frac{0,3010}{1,622} t = 2 - 1,903$$

$$\frac{0,3010}{1,622} t = 0,097 \quad / \cdot \frac{1,622}{0,3010}$$

$$t = 522,7 \text{ godina}$$

27. Dvije su lopte bačene istovremeno vertikalno prema gore. Prva ima početnu brzinu $v_1 = 20 \text{ m/s}$, a druga $v_2 = 24 \text{ m/s}$. Kolika je udaljenost između njih kada je prva lopta na maksimalnoj visini?
 A. 20.40 m B. 28.56 m C. 16.28 m D. 8.15 m E. 14.28

$$v_1 = 20 \text{ m/s}$$

Lopte su bačene istovremeno vertikalno prema gore.

$$v_2 = 24 \text{ m/s}$$

$$\Delta h = ?$$

Podaci za prvu bačenu loptu:

- maksimalna visina ili domet $H_1 = \frac{v_1^2}{2g} = \frac{(20 \text{ m/s})^2}{2 \cdot 9,81} = \frac{400 \text{ m}^2/\text{s}^2}{19,62 \text{ m/s}^2} = 20,387 \text{ m}$

- vrijeme utrošeno za postizane maksimalne visine $t_1 = \frac{v_1}{g} = \frac{20 \text{ m/s}}{9,81 \text{ m/s}^2} = 2,0387 \text{ sek} = 2,04 \text{ sek}$

Podaci za drugu bačenu loptu:

- prijeđeni put s_2 za vrijeme $t = 2,04 \text{ s}$ $s_2 = v_2 \cdot t - \frac{g}{2} t^2$

$$s_2 = 24 \text{ m/s} \cdot 2,04 \text{ s} - \frac{9,81 \text{ m/s}^2}{2} \cdot (2,04 \text{ s}) = 48,96 \text{ m} - 20,41 \text{ m} = 28,55 \text{ m}$$

Međusobna udaljenost Δh $\Delta h = s_2 - H_1 = 28,55 \text{ m} - 20,387 \text{ m} = 8,163 \text{ m}$
 odgovor D.

28. Koliki se rad izvrši ako se plinu početnog volumena 5 L uz stalan tlak $2 \cdot 10^5$ Pa povisi temperatura sa 27°C na 327°C ?

- A. 1256Nm B. 1000 J C. 725 J D. 1.52 J E. 910 J

izobarna promjena stanja plina; $p = \text{konstanta}$

$$V_1 = 5 \text{ L} = 5 \cdot 10^{-3} \text{ m}^3$$

$$p = 2 \cdot 10^5 \text{ Pa}$$

$$t_1 = 27^\circ\text{C}$$

$$T_1 = t_1 + 273 = 27^\circ\text{C} + 273 = 300 \text{ K}$$

$$t_2 = 327^\circ\text{C}$$

$$T_2 = t_2 + 273 = 327^\circ\text{C} + 273 = 600 \text{ K}$$

$$W = ?$$

Pomoću izobarne promjene stanja plina, izračunajmo krajnji volumen V_2 :

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad / \cdot T_1 T_2$$

$$V_1 T_2 = V_2 T_1 \quad \Rightarrow \quad V_2 = \frac{V_1 T_2}{T_1} = \frac{5 \cdot 10^{-3} \text{ m}^3 \cdot 600 \text{ K}}{300 \text{ K}} = 10 \cdot 10^{-3} \text{ m}^3$$

$$W = p \cdot (V_2 - V_1)$$

$$W = 2 \cdot 10^5 \text{ Pa} (10 \cdot 10^{-3} \text{ m}^3 - 5 \cdot 10^{-3} \text{ m}^3) = 2 \cdot 10^5 \text{ Pa} \cdot 5 \cdot 10^{-3} \text{ m}^3 = 10 \cdot 10^2 \text{ J} = 1000 \text{ J}$$

odgovor B.

29. Protoni se ubrzavaju u ciklotronu i udaraju u metu. Struja protonskog snopa na meti je $1.6 \mu\text{A}$. Koliko protona u jednoj sekundi udara u metu? ($e = 1.6 \cdot 10^{-19} \text{C}$)

A. 10^5 B. 30 000 C. 10^{12} D. 180 E. 10^{13}

$$I = 1,6 \mu\text{A} = 1,6 \cdot 10^{-6} \text{A}$$

$$e = 1,6 \cdot 10^{-19} \text{C}$$

$$t = 1 \text{sek}$$

$$n = N = ? \quad (\text{broj protona})$$

$$I = \frac{Q}{t} \quad / \cdot t$$

$$Q = I \cdot t = 1,6 \cdot 10^{-6} \text{A} \cdot 1 \text{s} = 1,6 \cdot 10^{-6} \text{C}$$

$$Q = n \cdot e$$

$$n = \frac{Q}{e} = \frac{1,6 \cdot 10^{-6} \text{C}}{1,6 \cdot 10^{-19} \text{C}} = 1 \cdot 10^{-6 - (-19)} = 10^{13}$$

odgovor E.

30. U trenutku kada se zamašnjak motora okreće s 60 okr/s, isključen je pogonski motor. Zamašnjak se zaustavi nakon 80 s. Koliki je put opisala za to vrijeme točka na obodu zamašnjaka?

A. 60π B. 6400π C. 4800π D. 900π E. 9600π

$$f = 60 \text{ okr/s} = 60 \text{ Hz}$$

$$\omega = 2 \cdot \pi \cdot f$$

$$\omega = \alpha \cdot t$$

$$t = 80 \text{ s}$$

$$\omega = 2 \cdot \pi \cdot 60$$

$$\alpha = \frac{\omega}{t} = \frac{120\pi}{80} = 1,5\pi \text{ s}^{-2}$$

$$\varphi(\text{kut}) = ?$$

$$\omega = 120\pi$$

$$\varphi = \frac{\alpha}{2} t^2 = \frac{1,5\pi}{2} \cdot 80^2 = 0,75\pi \cdot 6400 = 4800\pi$$

31. Kada se na oprugu objesi jedan uteg mase m , opruga se produlji za 11 cm. Koliko je titrajno vrijeme (period) dva utega (mase $2m$) kada titraju na toj istoj opruzi?

A. 60π B. 0.30 s C. 1.88 s D. 0.94 s E. 5.906 s

$$m_1 = m$$

$$m_2 = 2m$$

$$x_1 \text{ (produljenje opruge)} = 11 \text{ cm} = 0,11 \text{ m}$$

$$T_2 = ?$$

Kada uteg mase m visi na opruzi sila teže G jednaka je sili opruge F ;

$$F = k \cdot x_1 \quad G = m \cdot g \quad k = \text{konstanta opruge}$$

$$F = G$$

$$k \cdot x_1 = m \cdot g \quad /: x_1 \Rightarrow k = \frac{m \cdot g}{x_1} = \frac{m \text{ kg} \cdot 9,81 \text{ m/s}^2}{0,11 \text{ m}} = 89,18 \cdot m \frac{\text{N}}{\text{m}}$$

$$\text{Period titranja } T : T = 2\pi \sqrt{\frac{m}{k}}$$

$$T_2 = 2\pi \sqrt{\frac{m_2}{k}} = 2\pi \cdot \sqrt{\frac{2m}{89,18m}} = 2 \cdot 3,14 \cdot \sqrt{0,0224} = 6,28 \cdot 0,14975 = 0,94 \text{ s}$$

odgovor D.

32. Odredite smjer naboja i mase čestice koja gibajući se iz točke u kojoj potencijal iznosi 6000 V u točku s potencijalom 3400 V postigne brzinu od $5 \cdot 10^5$ m/s. Početna brzina čestice je nula.

- A. $1.7 \cdot 10^{11}$ C/kg B. $2.1 \cdot 10^{-8}$ C/kg C. 10^{-8} C/kg D. $9.6 \cdot 10^7$ C/kg
E. $4.8 \cdot 10^7$ C/kg

$$\begin{array}{l} \varphi_1 = 6000 \text{ V} \\ \varphi_2 = 3400 \text{ V} \end{array} \quad \left. \begin{array}{l} \Delta\varphi = \varphi_1 - \varphi_2 = 6000 \text{ V} - 3400 \text{ V} = 2600 \text{ V} \\ \Delta\varphi = U = 2600 \text{ V} = 2,6 \cdot 10^3 \text{ V} \end{array} \right\}$$

$$v = 5 \cdot 10^5 \text{ m/s}$$

$$v_0 = 0 \text{ m/s}$$

$$\frac{Q}{m} = ?$$

$$\left. \begin{array}{l} W = Q \cdot U \\ W = E_k = \frac{mv^2}{2} \end{array} \right\} \quad \begin{array}{l} W = E_k \\ Q \cdot U = \frac{mv^2}{2} \quad / \cdot 2 \end{array}$$

$$2Q \cdot U = mv^2$$

$$\frac{Q}{m} = \frac{v^2}{2 \cdot U}$$

$$\frac{Q}{m} = \frac{v^2}{2 \cdot U} = \frac{(5 \cdot 10^5 \text{ m/s})^2}{2 \cdot 2,6 \cdot 10^3 \text{ V}} = \frac{25 \cdot 10^{10} \text{ m}^2/\text{s}^2}{5,2 \cdot 10^3 \text{ V}} = 4,8 \cdot 10^7 \text{ C/kg}$$

33. Koliko kockica leda temperature $0\text{ }^{\circ}\text{C}$, stranice 2 cm , treba rastaliti u 1 l vode da bi ju ohladili s $26,5^{\circ}\text{C}$ na 10°C ? Specifična toplina taljenja leda je 333 kJkg^{-1} , specifični toplinski kapacitet vode je $4190\text{ Jkg}^{-1}\text{K}^{-1}$, gustoća vode je 10kgm^{-3} , a leda 920 kgm^{-3} . Gubitak topline u okolinu valja zanemariti!

A. 10 B. 15 C. 25 D. 20 E. 5

$$t_L (\text{temperatura leda}) = 0^{\circ}\text{C}$$

$$a (\text{stranica kockice leda}) = 2\text{ cm} = 2 \cdot 10^{-2}\text{ m}$$

$$\lambda (\text{specifična toplina taljenja}) = 333\text{ kJkg}^{-1} = 333\,000\text{ Jkg}^{-1} = 3,33 \cdot 10^5\text{ Jkg}^{-1}$$

$$c_v (\text{specifični toplinski kapacitet vode}) = 4190\text{ Jkg}^{-1}\text{K}^{-1}$$

$$\rho_v (\text{gustoća vode}) = 10^3\text{ kgm}^{-3}$$

$$\rho_L (\text{gustoća leda}) = 920\text{ kgm}^{-3}$$

$$V_v (\text{volumen vode}) = 1\text{ l} = 10^{-3}\text{ m}^3$$

$$\left. \begin{array}{l} t_1 (\text{voda}) = 26,5^{\circ}\text{C} \\ t_2 (\text{voda}) = 10^{\circ}\text{C} \end{array} \right\} \text{voda se hladi } \Delta t = t_1 - t_2 = 26,5^{\circ}\text{C} - 10^{\circ}\text{C} = 16,5^{\circ}\text{C} = 16,5\text{ K}$$

$$N (\text{broj kockica leda}) = ?$$

$$\text{Masa vode } m_v = \rho_v \cdot V_v = 10\text{ kgm}^{-3} \cdot 10^3\text{ m}^3 = 10^0\text{ kg} = 1\text{ kg}$$

$$Q_v = m_v \cdot c_v \cdot \Delta t = 1\text{ kg} \cdot 4190\text{ Jkg}^{-1}\text{K}^{-1} \cdot 16,5\text{ K} = 69\,135\text{ J}$$

$$Q_L = Q_v$$

$$Q_L = m_L \cdot \lambda \quad /: \lambda \quad \Rightarrow \quad m_L = \frac{Q_L}{\lambda} = \frac{69\,135\text{ J}}{3,33 \cdot 10^5\text{ Jkg}^{-1}} = 0,2\text{ kg}$$

Masa leda potrebnog za taljenje je $0,2\text{ kg}$. Izračunajmo masu jedne kockice leda m :

$$m = \rho_L \cdot V = \rho_L \cdot a^3 = 920\text{ kgm}^{-3} \cdot (2 \cdot 10^{-2}\text{ m})^3 = 920\text{ kgm}^{-3} \cdot 8 \cdot 10^{-6}\text{ m}^3 = 7,36 \cdot 10^{-3}\text{ kg}$$

Pomoću omjera potrebne mase leda izračunajmo poteban broj kockica:

$$N = \frac{m_L}{m} = \frac{0,2\text{ kg}}{7,36 \cdot 10^{-3}\text{ kg}} = 27 \quad \text{odgovor C.}$$

34. Pluteni čep liva napetroleju. Koliki je dio volumena čepa uronjen u petrolej ako gustoća pluta iznosi $0.2 \cdot 10^3 \text{ kg/m}^3$, a gustoća petroleja $0.8 \cdot 10^3 \text{ kg/m}^3$?

A. 0.4 B. 0.35 C. 0.3 D. 0.25 E. 0.2

$$V_p \text{ (volumen pluta)} = V$$

$$\rho_p \text{ (gustoća pluta)} = 0,2 \cdot 10^3 \text{ kg/m}^3$$

$$\rho_t \text{ (gustoća petroleja)} = 0,8 \cdot 10^3 \text{ kg/m}^3$$

$$V_u \text{ (volumen uronjenog dijela čepa)} = ?$$

$$G = U$$

$$V \cdot \rho_p \cdot g = V_u \cdot \rho_t \cdot g \quad /: (g \cdot \rho_t)$$

$$V_u = \frac{V \cdot \rho_p}{\rho_t} = \frac{V \cdot 0,2 \cdot 10^3 \text{ kg/m}^3}{0,8 \cdot 10^3 \text{ kg/m}^3}$$

$$V_u = 0,25 V$$

35. Dva jednaka pozitivna nabija iznosa $1\mu\text{C}$ smještena su u vakuumu u dva vrha jednakostraničnog trokuta stranice 0.5 m . Koliki je iznos električnog polja u trećem vrhu trokuta? ($\epsilon_0 = 8.854 \cdot 10^{-12}\text{ C}^2\text{N}^{-1}\text{m}^{-2}$)
 A. 57 kV/m B. 42.5 kV/m C. 62.3 kV/m D. 74.7 kV/m E. 72 kV/m

$$Q_1 = Q_2 = 1\mu\text{ C} = 10^{-6}\text{ C}$$

$$r = a(\text{stranica trokuta}) = 0,5\text{ m}$$

$$\epsilon_0 = 8.854 \cdot 10^{-12}\text{ C}^2\text{N}^{-1}\text{m}^{-2}$$

$$k = \frac{1}{4\pi\epsilon_0} = 9 \cdot 10^9\text{ Nm}^2\text{C}^{-2}$$

$$E = ?$$

$$E_1 = \frac{k \cdot Q_1}{r^2} = \frac{9 \cdot 10^9 \cdot 10^{-6}}{0,5^2} = \frac{9 \cdot 10^3}{25 \cdot 10^{-2}} = 3,6 \cdot 10^4\text{ V/m}$$

$$E_2 = \frac{k \cdot Q_2}{r^2} = \frac{9 \cdot 10^9 \cdot 10^{-6}}{0,5^2} = 3,6 \cdot 10^4\text{ V/m}$$

Iznos električnog polja u trećem vrhu predstavlja rezultanta E , koju možemo izračunati pomoću visine

$$\text{jednakostraničnog trokuta} \Rightarrow E = 2 \cdot v, \quad v = \text{visina trokuta} = \frac{a\sqrt{3}}{2},$$

$$a = E_1 = E_2 = 3,6 \cdot 10^4\text{ V/m} \quad \Rightarrow \quad v = \frac{a\sqrt{3}}{2} = \frac{E_1\sqrt{3}}{2} = \frac{3,6 \cdot 10^4 \cdot \sqrt{3}}{2}$$

$$E = 2 \cdot v = 2 \cdot \frac{3,6 \cdot 10^4 \cdot \sqrt{3}}{2} = 3,6 \cdot 10^4 \sqrt{3} = 62\,280\text{ V/m} = 62,28\text{ kV/m} = 62,3\text{ kV/m}$$

36. Koliki je polumjer zakrivljenosti udubljenog sfernog zrcala ako ono daje upola manju sliku predmeta, koji je od slike udaljen 85 cm?

A. 85 cm B. 170 cm C. 113 cm D. 57 cm E. 137 cm

x = udaljenost predmeta od zrcala

x' = udaljenost slike od zrcala

$$x - x' = 85 \text{ cm} \quad \Rightarrow \quad x = 85 + x'$$

$$y = -\frac{1}{2}$$

$$r(\text{polumjer zrcala}) = ?$$

$$y = -\frac{x'}{x}$$

$$x = 85 + x'$$

$$-\frac{1}{2} = -\frac{x'}{85 + x'} \quad / \cdot 2(85 + x')$$

$$x = 85 + 85$$

$$x = 170 \text{ cm}$$

$$85 + x' = 2x'$$

$$2x' - x' = 85$$

$$\frac{1}{x} + \frac{1}{x'} = \frac{1}{f}$$

$$x' = 85 \text{ cm}$$

$$\frac{1}{85} + \frac{1}{170} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{2+1}{170} = \frac{3}{170}$$

$$f = \frac{170}{3} = 56,67 \text{ cm}$$

$$r = 2f = 2 \cdot 56,67 \text{ cm} = 113,34 \text{ cm} \quad \text{odgovor C.}$$

37. Koliko naponskih članaka elektromotorne sile 1.5 V i unutrašnjeg otpora 0.25 Ω treba serijski povezati da bi strujnim krugom s vanjskim otporom od 10 Ω tekla struja jakosti 2 A?

A. 50 B. 20 C. 25 D. 40 E. 10

serijski spoj

$$E = 1,5 \text{ V}$$

$$R_u = 0,25 \text{ } \Omega$$

$$R_v = 10 \text{ } \Omega$$

$$I = 2 \text{ A}$$

$$n(\text{broj naponskih članaka}) = ?$$

$$I = \frac{n \cdot E}{n \cdot R_u + R_v} \quad / \cdot (nR_u + R_v)$$

$$I \cdot (nR_u + R_v) = n \cdot E$$

$$I \cdot n \cdot R_u + I \cdot n \cdot R_v = n \cdot E$$

$$I \cdot n \cdot R_u - n \cdot E = -I \cdot R_v$$

$$n(I \cdot R_u - E) = -I \cdot R_v \quad / : (I \cdot R_u - E)$$

$$n = \frac{-I \cdot R_v}{I \cdot R_u - E} = \frac{-I \cdot R_v}{-(E - I \cdot R_u)} = \frac{I \cdot R_v}{E - I \cdot R_u}$$

$$n = \frac{2 \text{ A} \cdot 10 \text{ } \Omega}{1,5 \text{ V} - 2 \text{ A} \cdot 0,25 \text{ } \Omega} = \frac{20}{1,5 - 0,5} = \frac{20}{1} = 20$$

odgovor B.

38. Od bakrenog štapa mase 1.5 kg želi se napraviti žica otpora 250 Ω. Kolika je duljina žice ako je električna otpornost bakra $1.7 \cdot 10^{-8} \Omega \text{ m}$, a gustoća bakra $8.9 \cdot 10^3 \text{ kg/m}^3$?
- A. 232 m B. 318 m C. 1.57 m D. 2.38 m E. 5.62 m

$$m = 1,5 \text{ kg}$$

$$R = 250 \Omega$$

$$\rho_{\text{Cu}} (\text{otpornost bakra}) = 1,7 \cdot 10^{-8} \Omega \text{ m}$$

$$\rho (\text{gustoća}) = 8,9 \cdot 10^3 \text{ kg/m}^3$$

$$l (\text{duljina žice}) = ?$$

|a

površina presjeka S $l = \text{duljina žice}$

|b

$$V = \text{volumen štapa} = L \cdot S$$

$$\rho = \frac{m}{V} \quad \Rightarrow \quad V = \frac{m}{\rho}$$

$$V = \frac{1,5 \text{ kg}}{8,9 \cdot 10^3 \text{ kg/m}^3} = 1,685 \cdot 10^{-4} \text{ m}^3$$

$$S = \frac{V}{l} = \frac{1,685 \cdot 10^{-4}}{l}$$

|c

$$R = \rho_{\text{Cu}} \cdot \frac{l}{S}$$

$$250 = 1,7 \cdot 10^{-8} \cdot \frac{l}{S}$$

$$250 = 1,7 \cdot 10^{-8} \cdot \frac{l}{\frac{1,685 \cdot 10^{-4}}{l}}$$

$$250 = 1,7 \cdot 10^{-8} \cdot \frac{l^2}{1,685 \cdot 10^{-4}}$$

$$250 = l^2 \cdot 1,008910^{-4}$$

$$\Rightarrow \quad l^2 = 2477946,278$$

$$l = \sqrt{2477946,278} = 1574 \text{ m} = 1,574 \text{ km}$$

39. U otvorena prazna kolica mase 800 kg , koja se gibaju horizontalno brzinom 1.5 m/s^2 , padne okomito odozgo 600 kg šljunka. Kolika će biti brzina kolica napunjenih šljunkom?
A. 1.50 m/s B. 2.00 m/s C. 1.02 m/s D. 0.86 m/s E. 0.63 m/s

$$m_1 \text{ (masa kolica) } = 800 \text{ kg}$$

$$v_1 \text{ (brzina kolica) } = 1,5 \text{ m/s}$$

$$m_2 \text{ (masa šljunka) } = 600 \text{ kg}$$

$$v_2 \text{ (početna brzina šljunka) } = 0 \text{ m/s}$$

$$v \text{ (zajednička brzina) } = ?$$

$$v = \frac{m_1 \cdot v_1 + m_2 \cdot v_2}{m_1 + m_2} = \frac{800 \text{ kg} \cdot 1,5 \text{ m/s} + 600 \text{ kg} \cdot 0 \text{ m/s}}{800 \text{ kg} + 600 \text{ kg}} = \frac{1200 \text{ kg m/s} + 0}{1400 \text{ kg}}$$

$$v = 0,857 \text{ m/s} \quad \text{odgovor D.}$$

40. Predmet miruje na horizontalnoj podlozi. Nakon što je dobio udarac u horizontalnom smjeru giba se 8 s i zaustavi se 32 m daleko od početnog položaja. Koliki je koeficijent trenja između predmeta i podloge?
- A. 0.051 B. 0.076 C. 0.102 D. 0.127 E.0.154

$$g = 9,81 \text{ m/s}^2$$

$$t = 8 \text{ s}$$

$$s = 32 \text{ m}$$

$$\mu(\text{koeficijent trenja}) = ?$$

Horizontalna sila koja je pomakla tijelo jednaka je sili trenja koja je zaustavila kolica:

$$F = F_r$$

$$m \cdot a = \mu \cdot m \cdot g$$

$$a = \mu \cdot g \quad \Rightarrow \quad \mu = \frac{a}{g}$$

Izračunajmo akceleraciju a:

$$s = \frac{a}{2} t^2 \quad \Rightarrow \quad a = \frac{2s}{t^2} = \frac{2 \cdot 32 \text{ m}}{(8 \text{ s})^2} = \frac{64 \text{ m}}{64 \text{ s}^2} = 1 \text{ m/s}^2$$

$$\text{koeficijent trenja:} \quad \mu = \frac{a}{g} = \frac{1 \text{ m/s}^2}{9,81 \text{ m/s}^2} = 0,102 \quad \text{odgovor C.}$$

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