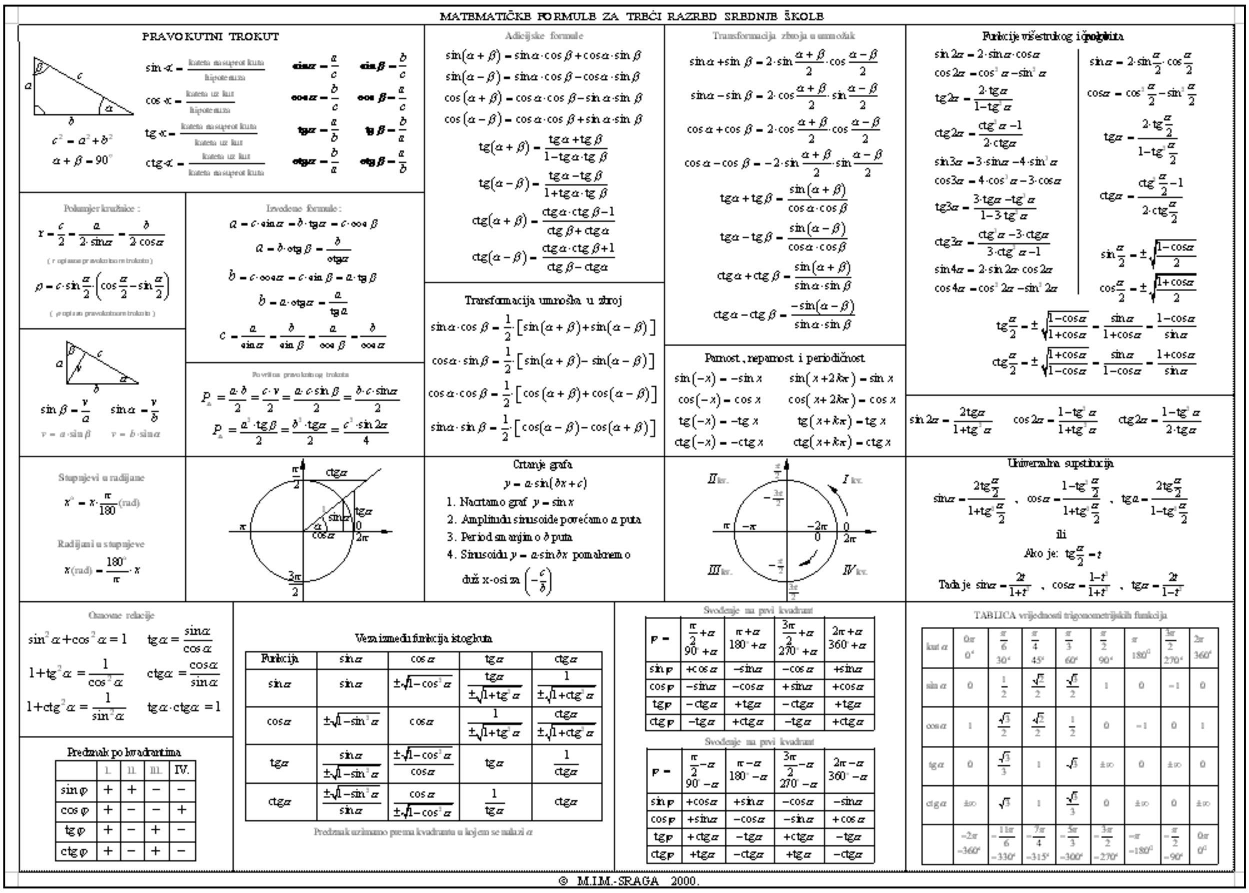
POTPUNO RIJEŠENI ZADACI

PRIRUČNIK ZA SAMOSTALNU PRIPREMU
PRIJEMNOG ISPITA
NA

TEHNIČKE FAKULTETE

2000/2001.



Zadatke riješili i grafički obradili * IVANA i MLADEN SRAGA *

Zadaci su uzeti iz matematičko fizičkog lista .

Zadatke riješili:
IVANA SRAGA
MLADEN SRAGA

Grafička obrada: **MLADEN SRAGA**
Matematički slog: **MLADEN SRAGA**

Tisak za vlastite potrebe
M.I.M.-Sraga d.o.o.

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2000./2001.g.

M-1. Izraz $\frac{(a+3)^6-(a+3)^4}{(a+2)^4-(a+2)^2}\cdot\frac{(a+1)^3-(a+1)}{(a+3)^5-(a+3)^3}$ jednak je

A. $\frac{a+3}{a+1}$ B. $\frac{a}{a+1}$ C. $\frac{a}{a+2}$ D. $\frac{a+3}{a+2}$ E. $\frac{(a+3)a}{a+2}$

$$\frac{(a+3)^6-(a+3)^4}{(a+2)^4-(a+2)^2}\cdot\frac{(a+1)^3-(a+1)}{(a+3)^5-(a+3)^3}=$$

$$=\frac{(a+3)^4\cdot\left[(a+3)^2-1\right]}{(a+2)^2\cdot\left[(a+2)^2-1\right]}\cdot\frac{(a+1)\cdot\left[(a+1)^2-1\right]}{(a+3)^3\cdot\left[(a+3)^2-1\right]}=$$ Kratimo kockaste zagrade $=$

$$=\frac{(a+3)^4\cdot(a+1)\left[(a+1)^2-1\right]}{(a+3)^3\cdot(a+2)^2\cdot\left[(a+2)^2-1\right]}=$$

$$=\frac{(a+3)\cdot(a+1)\cdot(a+1-1)\cdot(a+1+1)}{(a+2)^2\cdot(a+2-1)\cdot(a+2+1)}=$$

$$=\frac{(a+3)\cdot(a+1)\cdot a\cdot(a+2)}{(a+2)^2\cdot(a+1)\cdot(a+3)}=$$

$$=\frac{a}{a+2}$$

ALGEBARSKI IZRAZI

$$(a+b)^2=(a+b)\cdot(a+b)=a^2+2ab+b^2$$
$$(a+b)^2=(b+a)^2$$
$$(a-b)^2=(a-b)\cdot(a-b)=a^2-2ab+b^2$$
$$(a-b)^2=(b-a)^2$$
$$(-a-b)^2=(a+b)^2$$
$$(a-b)\cdot(a+b)=a^2-b^2$$
$$(a+b)^3=a^3+3a^2b+3ab^2+b^3$$
$$(a-b)^3=a^3-3a^2b+3ab^2-b^3$$
$$a^3-b^3=(a-b)\cdot(a^2+ab+b^2)$$
$$a^3+b^3=(a+b)\cdot(a^2-ab+b^2)$$
$$(a+b+c)^2=a^2+b^2+c^2+2ab+2ac+2bc$$
$$x^2+px+q=\left\{\begin{matrix}m+n=p\\ m\cdot n=q\end{matrix}\right\}=(x+m)\cdot(x+n)$$
$$ax^2+bx+c=\left\{\begin{matrix}m+n=b\\ m\cdot n=a\cdot c\end{matrix}\right\}=ax^2+mx+nx+c=\dots$$

M-2. Za kvadratnu funkciju $f(x) = ax^2 + bx + c$ poznato je da vrijedi $f(-2) = 3$, $f(0) = 1$ i $f(2) = -3$. Tada $f(1)$ iznosi :

A. $-\frac{1}{4}$ B. $-\frac{3}{4}$ C. 2 D. $\frac{3}{4}$ E. $\frac{3}{2}$

$f(x) = ax^2 + bx + c$

$f(-2) = 3 \rightarrow f(x) = 3$, $\downarrow$ $x = -2$

$$f(x) = ax^2 + bx + c$$
$$3 = a\cdot(-2)^2 + b\cdot(-2) + c$$
$$3 = 4a - 2b + c$$

za $f(0) = 1 \rightarrow f(x) = 1$, $\downarrow$ $x = 0$

$$f(x) = ax^2 + bx + c$$
$$1 = a\cdot 0^2 + b\cdot 0 + c$$
$$1 = 0 + 0 + c$$
$$c = 1$$

za $f(2) = -3 \rightarrow f(x) - 3$, $\downarrow$ $x = 2$

$$f(x) = ax^2 + bx + c$$
$$-3 = a\cdot 2^2 + b\cdot 2 + c$$
$$-3 = 4a + 2b + c$$

Sada riješimo sustav

$$4a - 2b + c = 3$$
$$4a + 2b + c = -3$$
$$c = 1$$

$$\left.\begin{array}{l} 4a - 2b + 1 = 3 \\ 4a + 2b + 1 = -3 \end{array}\right\} +$$

$$8a + 2 = 0$$
$$8a = -2 \quad / :8$$
$$a = -\frac{2}{8}$$
$$a = -\frac{1}{4}$$

$$4a - 2b + 1 = 3 \quad , \quad a = -\frac{1}{4}$$
$$4\cdot\left(-\frac{1}{4}\right) - 2b = 3 - 1$$
$$-1 - 2b = 2$$
$$-2b = 2 + 1$$
$$-2b = 3 \quad / :(-2)$$
$$b = -\frac{3}{2}$$

$$f(x) = ax^2 + bx + c$$
$$f(x) = -\frac{1}{4}x^2 = \frac{3}{2}x + 1$$

Pa je: $f(1) = -\frac{1}{4}\cdot 1^2 - \frac{3}{2}\cdot 1 + 1 = -\frac{1}{4} - \frac{3}{2} + 1$

$$f(1) = \frac{-1 - 6 + 4}{4} = -\frac{3}{4}$$

M-3. Za koje realne brojeve x je $\sqrt{3x^3 + x^2 - 2x}$ realan broj?

A. $x \in [-1, 0] \cup \left[\frac{2}{3}, \infty\right)$ B. $x \geq -1$ C. $x \in \left[0, \frac{2}{3}\right]$ D. $x \leq \frac{2}{3}$ E. $x \in \left[-1, \frac{2}{3}\right]$

Zbog korjena mora biti: $3x^3 + x^2 - 2x \geq 0$

$$x\left(3x^2 + x - 2\right) \geq 0$$

I $(+,+)$

$x \geq 0,$ $3x^2 + x - 2 \geq 0$

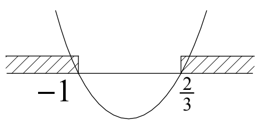
$$x_{1,2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 3 \cdot (-2)}}{2 \cdot 3}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 + 24}}{6}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{25}}{6}$$

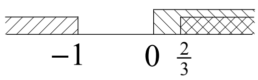
$$x_1 = \frac{-1 + 5}{6} = \frac{4}{6} = \frac{2}{3}$$

$$x_2 = \frac{-1 - 5}{6} = \frac{-6}{6} = -1$$



$x \geq 0$ $x \in [-\infty, -1] \cup \left[\frac{2}{3}, \infty\right\rangle$

$\cap$



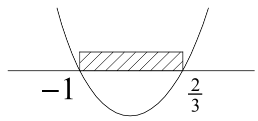
$$x \in \left[\frac{2}{3}, \infty\right\rangle$$

II $(-,-)$

$x \leq 0,$ $3x^2 + x - 2 \leq 0$

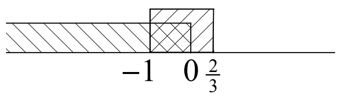
$$x_1 = \frac{2}{3}$$

$$x_2 = -1$$



$x \leq 0$ $x \in \left[1, \frac{2}{3}\right]$

$\cap$



$$x \in [-1, 0]$$

$\cup$

$$x \in [-1, 0] \cup \left[\frac{2}{3}, \infty\right\rangle$$

M-4. Ako je $f(x)=3x-1, g(x)=x^3$ i $h(x)=\sqrt{x}$, onda vrijednost funkcije $f\circ(g\circ h)$ u točki $3^{-\frac{2}{3}}$ iznosi:

A. -1 B. 2 C. 1 D. 0 E. -2

$f(x)=3x-1$, $g(x)=x^3$, $h(x)=\sqrt{x}$

$f\circ(g\circ h)=?$

Uvedemo $w=g\circ h=\sqrt{x^3}=\left(x^3\right)^{\frac{1}{2}}=x^{\frac{3}{2}}$

Pa je sada:

$f\circ(g\circ h)=f\circ w=3\cdot x^{\frac{3}{2}}-1$

$f\circ(g\circ h)$ za $3^{-\frac{2}{3}}=3\cdot\left(3^{-\frac{2}{3}}\right)^{\frac{3}{2}}-1=3\cdot 3^{-1}-1=3\cdot\frac{1}{3}-1=1-1=0$

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M-5. Ako je $3^{2x+y}=\frac{1}{\sqrt{27}}$ i $3^{-x+y}=\sqrt{3}$, onda je izraz $x+2y$ jednak

A. -2 B. -1 C. 0 D. 1 E. 2

$$3^{2x+y}=\frac{1}{\sqrt{27}}$$
$$3^{2x+y}=\frac{1}{\left(3^3\right)^{\frac{1}{2}}}$$
$$3^{2x+y}=3^{-\frac{3}{2}}$$
$$2x+y=-\frac{3}{2}$$

$$3^{-x+y}=\sqrt{3}$$
$$3^{-x+y}=3^{\frac{1}{2}}$$
$$-x+y=\frac{1}{2}$$

sustav:

$$2x+y=-\frac{3}{2}$$
$$-x+y=\frac{1}{2}\quad /\cdot 2$$

$$\left.\begin{aligned}2x+y&=-\frac{3}{2}\\-2x+2y&=1\end{aligned}\right\}+$$

$$3y=1-\frac{3}{2}$$
$$3y=\frac{2-3}{2}$$
$$3y=-\frac{1}{2}\quad /\cdot\frac{1}{3}$$
$$y=-\frac{1}{6}$$

$$2x+y=-\frac{3}{2}\quad,\qquad y=-\frac{1}{6}$$
$$2x-\frac{1}{6}=-\frac{3}{2}$$
$$2x=\frac{1}{6}-\frac{3}{2}$$
$$2x=\frac{1-9}{6}$$
$$2x=-\frac{8}{6}$$
$$2x=-\frac{4}{3}\quad /\cdot\frac{1}{2}$$
$$x=-\frac{2}{3}$$

Traženi izraz: $x+2y=-\frac{2}{3}+2\cdot\left(-\frac{1}{6}\right)=-\frac{2}{3}-\frac{1}{3}=-\frac{3}{3}=-1$

M-6. Ako je $\log a + 3\log b = 2$ i $\log a - \log b = 1$, onda je izraz $a \cdot b$ jednak
A. $\sqrt{10}$ B. 10 C. $10\sqrt{10}$ D. 100 E. $100\sqrt{10}$

$$\log a + 3\log b = 2$$
$$\log a - \log b = 1 \quad / \cdot 3$$

$$\left.\begin{aligned} \log a + 3\log b &= 2 \\ 3\log a - 3\log b &= 3 \end{aligned}\right\} +$$

$$4\log a = 5 \quad / :4$$
$$\log a = \frac{5}{4}$$

Po pravilu za antilogaritmiranje: $\log_A B = C \Rightarrow B = A^C$

$$a = 10^{\frac{5}{4}}$$

$$\log a - \log b = 1 \quad , \quad \log a = \frac{5}{4}$$
$$\frac{5}{4} - \log b = 1$$
$$\frac{5}{4} - 1 = \log b$$
$$\log b = \frac{1}{4}$$

Po pravilu za antilogaritmiranje: $\log_A B = C \Rightarrow B = A^C$

$$b = 10^{\frac{1}{4}}$$

Pa je: $a \cdot b = 10^{\frac{5}{4}} \cdot 10^{\frac{1}{4}} = 10^{\frac{6}{4}} = 10^{\frac{3}{2}} = \left(10^3\right)^{\frac{1}{2}} = \sqrt{10^3} = \sqrt{10^2 \cdot 10^1} = 10\sqrt{10}$

$\log_a b = c \Rightarrow b = a^c$	(1)	$\log_{\frac{1}{a}} b = -\log_a b$	(9)
$\log(a \cdot b) = \log a + \log b$	(2)	$\log_a b = \frac{\log b}{\log a}$ ili $\log_a b = \frac{\log_c b}{\log_c a}$	(10)
$\log \frac{a}{b} = \log a - \log b$	(3)	$\log_{a^n} b = \frac{1}{n}\log_a b$	(11)
$\log a^x = x \cdot \log a$	(4)	$a^{\log_a b} = b$	(12)
$\log \sqrt[n]{a} = \log a^{\frac{1}{n}} = \frac{1}{n}\log a$	(5)	$\log x = \log y$ $x = y$	(13)
$\log \sqrt{a} = \log a^{\frac{1}{2}} = \frac{1}{2}\log a$	(5A)	Logaritamske nejednadžbe $\log_c a > \log_c b$ I: ako je $c > 1$ tada je $a > b$ II: ako je $0 < c < 1$ tada je $a < b$	(14)
$\log_a a = 1$ (6) $\quad \log_a\left(a^x\right) = x$	(6A)		
$\log_a 1 = 0$	(7)	$\log_c a < \log_c b$ I: ako je $c > 1$ tada je $a < b$ II: ako je $0 < c < 1$ tada je $a > b$	(15)
$\log_a b = \frac{1}{\log_b a}$	(8)		

M-7. Zbroj rješenja jednadžbe $5^{2x+1}+4^{x+\frac{1}{2}}=7\cdot 10^{x}$ iznosi:

A. -1 B. 0 C. $\frac{5}{2}$ D. $\frac{7}{2}$ E. $\frac{3}{2}$

$$5^{2x+1}+4^{x+\frac{1}{2}}=7\cdot 10^{x} \quad \Big/ \cdot\frac{1}{10^{x}}$$

$$\frac{5^{2x}\cdot 5^{1}}{10^{x}}+\frac{4^{x}\cdot 4^{\frac{1}{2}}}{10^{x}}=7$$

$$\frac{\left(5^{2}\right)^{x}\cdot 5}{10^{x}}+\frac{4^{x}\cdot\sqrt{4}}{10^{x}}=7$$

$$\frac{25^{x}\cdot 5}{10^{x}}+\frac{4^{x}\cdot 2}{10^{x}}=7$$

$$5\cdot\left(\frac{25}{10}\right)^{x}+2\cdot\left(\frac{4}{10}\right)^{x}-7=0$$

$$5.\left(\frac{5}{2}\right)^{x}+2\cdot\left(\frac{2}{5}\right)^{x}-7=0$$

$$5\cdot\left(\frac{5}{2}\right)^{x}+2\cdot\left(\left(\frac{5}{2}\right)^{x}\right)^{-1}-7=0$$

uvedemo: $t=\left(\frac{5}{2}\right)^{x}$

$$5\cdot t+2\cdot t^{-1}-7=0$$

$$5\cdot t+\frac{2}{t}-7=0 \quad \Big/ \cdot t$$

$$5\cdot t^{2}+2-7\cdot t=0$$

$$5\cdot t^{2}-7\cdot t+2=0$$

$$t_{1,2}=\frac{-(-7)\pm\sqrt{(-7)^{2}-4\cdot 5\cdot 2}}{2\cdot 5}=\frac{7\pm\sqrt{49-40}}{10}$$

$$t_{1,2}=\frac{7\pm\sqrt{9}}{10}=\frac{7\pm 3}{10}$$

$$t_{1}=\frac{7+3}{10}=\frac{10}{10}=1 \qquad t_{2}=\frac{7-3}{10}=\frac{4}{10}=\frac{2}{5}$$

$$t=\left(\frac{5}{2}\right)^{x}$$

$$\left(\frac{5}{2}\right)^{x}=1 \qquad \left(\frac{5}{2}\right)^{x}=\frac{2}{5}$$

$$\left(\frac{5}{2}\right)^{x}=\left(\frac{5}{2}\right)^{0} \qquad \left(\frac{5}{2}\right)^{x}=\left(\frac{5}{2}\right)^{-1}$$

$$x=0 \qquad x=-1$$

$$x_{1}=0 \qquad x_{2}=-1$$

Zbroj rješenja je: $x_{1}+x_{2}=0+(-1)=-1$

M-8. Zbroj 30 uzastopnih parnih prirodnih brojeva iznosi 1230. Najveći od njih je

A. 62 B. 64 C. 66 D. 68 E. 70

Ovdje se radi o Aritmetičkom nizu, jer je $d = 2$ -razlika dvaju susjednih parnih brojeva...

Prvi parni broj označimo sa $2x$ - taj broj je paran bez obzira na x jer se množi sa dva ...

$$a_1 = 2x$$

$$a_2 = 2x+2 \quad \rightarrow \quad a_2 = a_1 + d$$

$$a_3 = 2x+4 \quad \rightarrow \quad a_3 = a_1 + 2\cdot d$$

$$\left.\begin{matrix} a_2 = a_1 + d \\ a_3 = a_1 + 2\cdot d \end{matrix}\right\} d = a_2 - a_1 = 2x+2-2x = 2$$

$$S_{30} = 1230 \ , \ n = 30 \ , \ d = 2$$

$$S_n = \frac{n}{2}\cdot\left[2\cdot a_1 + (n-1)\cdot d\right]$$

$$1230 = \frac{30}{2}\cdot\left[2\cdot 2x + (30-1)\cdot 2\right]$$

$$1230 = 15\cdot(4x + 29\cdot 2) \quad \Big/ \cdot\frac{1}{15}$$

$$82 = 4x + 58$$

$$82 - 58 = 4x$$

$$4x = 24 \quad / :4$$

$$x = 6 \qquad a_1 = 2x = 2\cdot 6$$

$$a_1 = 12$$

Traženi najveći broj je: $a_{30} = a_1 + 29\cdot d$

$$a_{30} = 12 + 29.2 = 12 + 58$$

$$a_{30} = 70$$ → Rješenje pod: E

	Opći član	Diferencija	Zbroj prvih n članova	Interpolacija
Aritmetički niz $a_2 = \frac{a_1+a_3}{2}$ $a_n = \frac{a_{n-1}+a_{n+1}}{2}$	$a_n = a_1 + (n-1)\cdot d$ $n = \frac{a_n - a_1 + d}{d}$	$d = a_2 - a_1 = a_3 - a_2$ $d = a_n - a_{n-1}$	$S_n = \frac{n}{2}\cdot(a_1 + a_n)$ $S_n = \frac{n}{2}\cdot\left[2\cdot a_1 + (n-1)\cdot d\right]$	$\delta = \frac{b-a}{r+1}$
	Opći član	Kvocijent	Zbroj prvih n članova	Interpolacija
Geometrijski niz $a_2^2 = a_1\cdot a_3$ $a_n^2 = a_{n-1}\cdot a_{n+1}$	$a_n = a_1\cdot q^{n-1}$	$q = \frac{a_2}{a_1} = \frac{a_3}{a_2}$ $q = \frac{a_n}{a_{n-1}}$	$S_n = a_1\cdot\frac{q^n-1}{q-1}$ $S_n = \frac{a_n\cdot q - a_1}{q-1}$	$q' = \sqrt[r+1]{\frac{b}{a}}$

M-9. Prilikom rješavanja jednog zadatka iz matematike, 12% učenika nije riješilo zadatak, 32% učenika je djelomično riješilo zadatak, a ostatak od 14 učenika je zadatak točno riješilo. Koliko je učenika bilo u razredu?

A. 21 B. 22 C. 23 D. 24 E. 25

12% – nije riješilo zadatak
32% – je rješilo dio
14 – učenika je riješilo zadatak

Postotak učenika koji je riješio zadatak je:

$$100\% - (12\% + 32\%) = 100\% - 44\% = 56\%$$

Sada znamo da je 14 učenika $= 56\%$ od ukupnog broja učenika u razredu.

Pišemo: 56% od $x = 14$ učenika, x = ukupan broj učenika

ili $56\% \cdot x = 14$

$$\frac{56}{100} \cdot x = 14 \qquad / \cdot \frac{100}{56}$$

$$x = 14 \cdot \frac{100}{56}$$

$$x = 25$$ → Rješenje pod: E

M-10. Površina trokuta, kojemu je duljina jedne stranice 4, a kutovi uz tu stranicu 45° i 60°, jednaka je

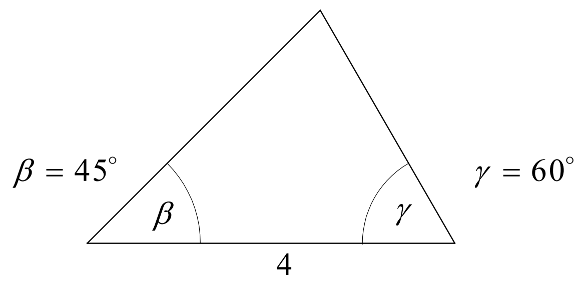
A. $4\left(3-\sqrt{3}\right)$ E. $6\left(\sqrt{6}-2\right)$ C. $2\left(\sqrt{5}-1\right)$ D. $8\left(\sqrt{3}-1\right)$ E. $9\left(\sqrt{2}-1\right)$

$a = 4$

$\beta = 45^\circ$

$\gamma = 60^\circ$

$P = ?$



$\alpha = 180^\circ - (\beta + \gamma)$

$\alpha = 180^\circ - \left(45^\circ + 60^\circ\right)$

$\alpha = 180^\circ - 105^\circ$

$\alpha = 75^\circ$

$$P = \frac{a^2 \cdot \sin\beta \cdot \sin\gamma}{\sin\alpha} \qquad \sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$$

$$P = \frac{4^2 \cdot \sin 45^\circ \cdot \sin 60^\circ}{\sin 75^\circ} \quad \rightarrow \quad \sin 75^\circ = \sin\left(30^\circ + 45^\circ\right) = \updownarrow \text{ po gornjoj formuli}$$

$$= \sin 30^\circ \cdot \cos 45^\circ + \cos 30^\circ \cdot \sin 45^\circ =$$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2\cdot 2}\cdot\left(1+\sqrt{3}\right)$$

$$= \frac{\sqrt{2}}{4}\cdot\left(\sqrt{3}+1\right)$$

$$P = \frac{16\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}}{2\cdot\frac{\sqrt{2}}{4}\cdot\left(\sqrt{3}+1\right)}$$

$$P = \frac{4\cdot\sqrt{2}\cdot\sqrt{3}}{\frac{\sqrt{2}}{2}\cdot\left(\sqrt{3}+1\right)} = \frac{\frac{4\cdot\sqrt{2}\cdot\sqrt{3}}{1}}{\frac{\sqrt{2}\cdot\left(\sqrt{3}+1\right)}{2}} = \frac{2\cdot 4\cdot\sqrt{2}\cdot\sqrt{3}}{\sqrt{2}\cdot\left(\sqrt{3}+1\right)} = \frac{8\cdot\sqrt{3}}{\sqrt{3}+1}$$

$$P = \frac{8\cdot\sqrt{3}}{\sqrt{3}+1}\cdot\frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{8\cdot\sqrt{3}^2 - 8\sqrt{3}}{\sqrt{3}^2 - 1^2}$$

$$P = \frac{8\cdot 3 - 8\sqrt{3}}{3-1} = \frac{8\cdot\left(3-\sqrt{3}\right)}{2}$$

$$P = \frac{2\cdot 4\cdot\left(3-\sqrt{3}\right)}{2}$$

$P = 4\cdot\left(3-\sqrt{3}\right)$ $\rightarrow$ Rješenje pod: A

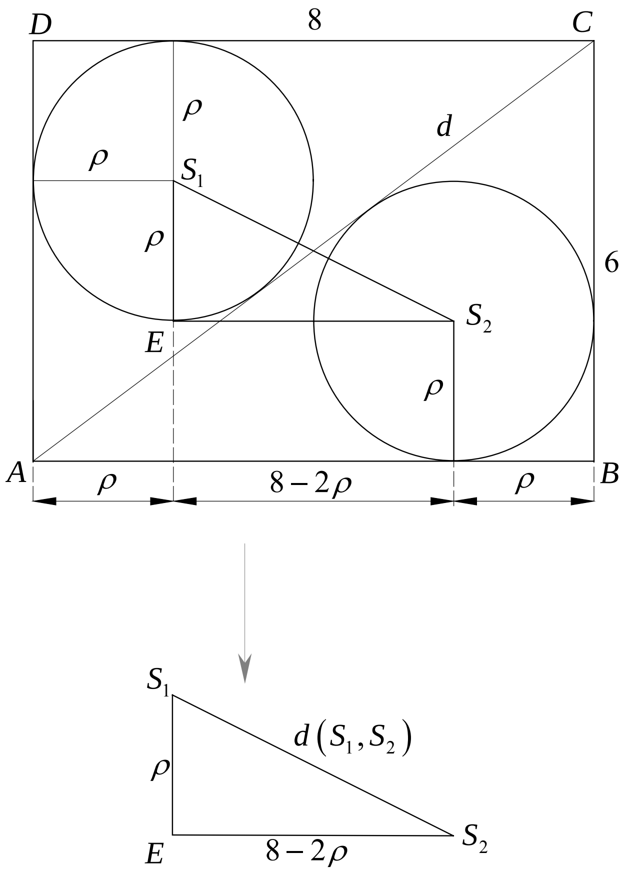
Adicijske formule

$$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$$
$$\operatorname{tg}(\alpha+\beta) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1-\operatorname{tg}\alpha\cdot\operatorname{tg}\beta}$$
$$\operatorname{tg}(\alpha-\beta) = \frac{\operatorname{tg}\alpha - \operatorname{tg}\beta}{1+\operatorname{tg}\alpha\cdot\operatorname{tg}\beta}$$
$$\operatorname{ctg}(\alpha+\beta) = \frac{\operatorname{ctg}\alpha\cdot\operatorname{ctg}\beta - 1}{\operatorname{ctg}\beta + \operatorname{ctg}\alpha}$$
$$\operatorname{ctg}(\alpha-\beta) = \frac{\operatorname{ctg}\alpha\cdot\operatorname{ctg}\beta + 1}{\operatorname{ctg}\beta - \operatorname{ctg}\alpha}$$

M-11. Pravokutnik *ABCD* čije stranice imaju duljine 6 i 8, podijeljen je dijagonalom na trokute $\triangle ABC$ i $\triangle CDA$. Udaljenost između središta kružnica upisanih u te trokute iznosi:

A. $\frac{13}{3}$ B. $2\sqrt{5}$ C. $\frac{5\sqrt{3}}{2}$ D. $3\sqrt{2}$ E. $\frac{9}{2}$

Nacrtajmo sliku:



Treba uočiti pravokutan trokut S_1ES_2 - čija je hipotenuza jednaka $d(S_1, S_2)$

1. Izračunamo površinu trokuta *ABC*

$$P = \frac{8\cdot 6}{2}$$
$$P = 4\cdot 6$$
$$P = 24$$

2. $d^2 = 8^2 + 6^2$ $s = \frac{a+b+d}{2}$

$d^2 = 100$ $s = \frac{8+6+10}{2}$

$d = 10$ $s = 12$

3. Površina trokuta jednaka je i:

$P = \rho\cdot s$ uvrstimo sve poznato:

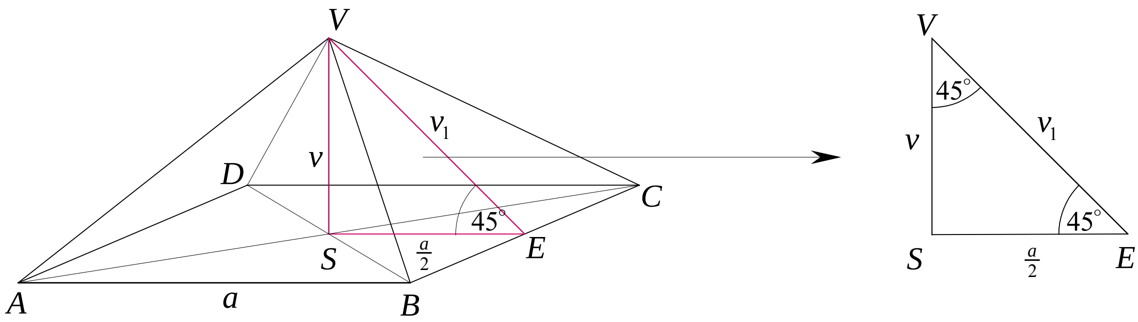
$$24 = \rho\cdot 12$$
$$12\cdot\rho = 24$$
$$\rho = 2$$

4.
$$d(S_1,S_2)^2 = \rho^2 + (8-2\rho)^2$$
$$d(S_1,S_2)^2 = 2^2 + (8-2\cdot 2)^2$$
$$d(S_1,S_2)^2 = 4 + 4^2$$
$$d(S_1,S_2)^2 = 4 + 16$$
$$d(S_1,S_2)^2 = 20$$
$$d(S_1,S_2) = \sqrt{20}$$
$$d(S_1,S_2)^2 = \sqrt{4\cdot 5}$$
$$d(S_1,S_2)^2 = 2\sqrt{5}$$

M-12. Volumen uspravne četverostrane piramide kojoj je baza kvadrat stranice duljine a, a bočne strane nagnute pod kutem od 45° u odnosu na bazu iznosi:

A. $\frac{a^2}{2}$ B. $\frac{a^2\sqrt{2}}{2}$ C. $\frac{a}{2}$ D. $a^3\sqrt{2}$ E. $\frac{a^3}{6}$

Nacrtajmo skicu te piramide, pa izdvojimo pravokutni trokut ESV ...
Kut što ga zatvara bočna stranica piramide s bazom je ustvari
kut koji zatvara visina bočne stranice (v_1) s bazom...



Izdvojimo pravokutan trokut *ESV*

$$\mathrm{tg}\sphericalangle = \frac{\text{kateta nasuprot kuta}}{\text{kateta uz kut}}$$

$$\mathrm{tg}\,45^\circ = \frac{v}{\frac{a}{2}}$$

$$1 = \frac{\frac{v}{1}}{\frac{a}{2}}$$

$$1 = \frac{2\cdot v}{a} \quad / \cdot a$$

$$a = 2\cdot v \quad / :2$$

$$v = \frac{a}{2}$$

$$V = \frac{B\cdot v}{3} \quad , \quad B = a^2$$

$$V = \frac{a^2\cdot v}{3}$$

$$V = \frac{a^2.\frac{a}{2}}{3}$$

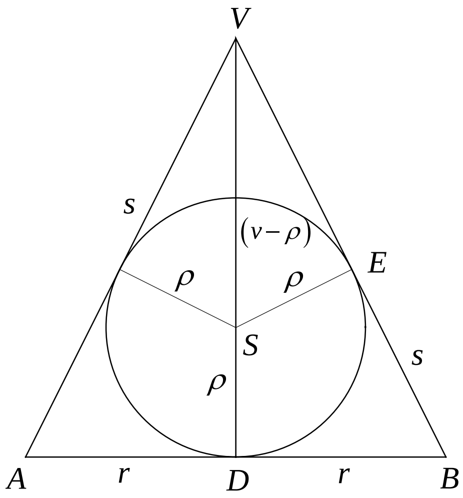
$$V = \frac{\frac{a^3}{2}}{\frac{3}{1}}$$

$$V = \frac{a^3}{6}$$

M-13. Volumen kugle upisane u stožac polumjera baze $r = 3$ i visine $v = 4$ iznosi:

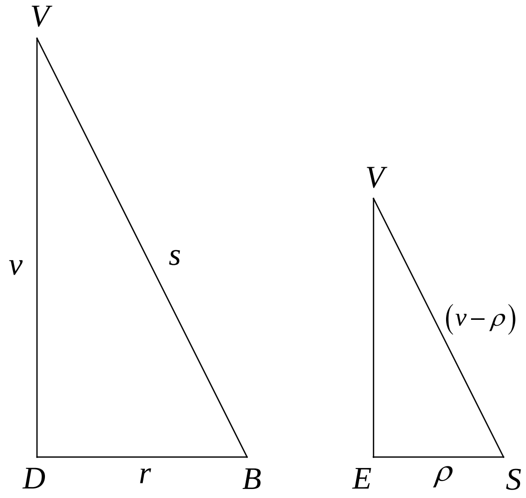
A. 4π B. $\frac{3\sqrt{2}}{2}\pi$ C. $\frac{5}{2}\pi$ D. $2\sqrt{5}\pi$ E. $\frac{9}{2}\pi$

Nacrtajmo skicu:



Treba uočiti slične trokute BDV i SEV

↓



$$s^2 = r^2 + v^2$$
$$s^2 = 3^2 + 4^2$$
$$s^2 = 9 + 16$$
$$s^2 = 25$$
$$s = 5$$

Kako su trokuti BDV i SEV slični stranice im se odnose:

$$s : (v - \rho) = r : \rho$$
$$5 : (4 - \rho) = 3 : \rho$$
$$\frac{5}{4-\rho} = \frac{3}{\rho} \quad / \cdot (4-\rho) \cdot \rho$$
$$5 \cdot \rho = 3 \cdot (4 - \rho)$$
$$5\rho = 12 - 3\rho$$
$$5\rho + 3\rho = 12$$
$$8\rho = 12$$
$$\rho = \frac{12}{8}$$
$$\rho = \frac{3}{2}$$

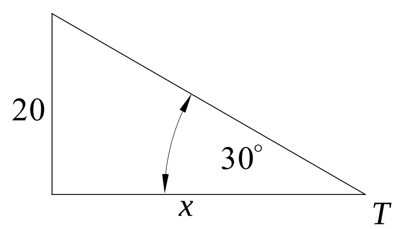
Volumen kugle je:

$$V = \frac{4}{3} \cdot \rho^3 \cdot \pi$$
$$V = \frac{4}{3} \cdot \left(\frac{3}{2}\right)^3 \cdot \pi$$
$$V = \frac{4}{3} \cdot \frac{3^3}{2^3} \cdot \pi$$
$$V = \frac{4}{3} \cdot \frac{3 \cdot 9}{4 \cdot 2} \cdot \pi$$
$$V = \frac{9}{2}\pi$$

M-14. Toranj visok 20 *m* vidi se pod kutem od 30°, iz točke koja leži u ravnini podnožja tornja. Dvostruko viši toranj vidio bi se iz iste točke pod kutem od

A. $48^\circ 3'$ B. $46^\circ 8'$ C. $49^\circ 6'$ D. $50^\circ 4'$ E. $51^\circ 2'$

Nacrtajmo skicu:



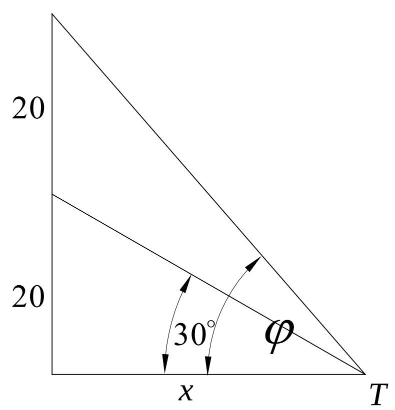
1. Sa x označimo udaljenost točke T od tornja...

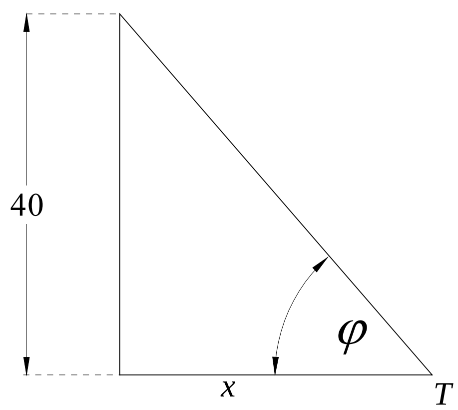
$$\operatorname{tg} 30^\circ = \frac{20}{x} \quad / \cdot \frac{x}{\operatorname{tg} 30^\circ}$$

$$x = \frac{20}{\operatorname{tg} 30^\circ}$$

$$x = \frac{20}{\frac{\sqrt{3}}{3}} = \frac{\frac{20}{1}}{\frac{\sqrt{3}}{3}} = \frac{60}{\sqrt{3}} = \frac{60}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3}$$

$$x = 20\sqrt{3}$$





Nacrtamo sada dvostruko viši toranj i sa φ označimo traženi kut...

$$\operatorname{tg}\varphi = \frac{\text{kateta nasuprot kuta}}{\text{kateta uz kut}}$$

$$\operatorname{tg}\varphi = \frac{40}{x}$$

$$\operatorname{tg}\varphi = \frac{40}{20\sqrt{3}} = \frac{40}{20 \cdot 1,732} = \frac{40}{34,641}$$

$$\operatorname{tg}\varphi = 1,1547 \quad / \operatorname{tg}^{-1}$$

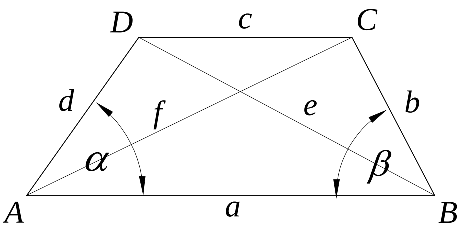
$$\varphi = 49^\circ 06' 24''$$

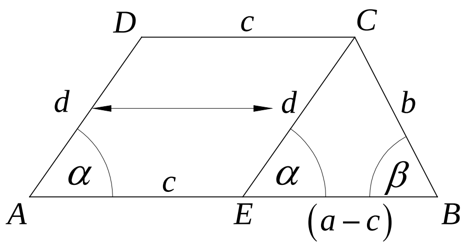
$$\varphi = 49^\circ 6'$$

M-15. U trapezu su poznate duljine osnovica $a = 6$ cm i $c = 4$ cm, te duljine krakova $b = 3$ cm i $d = 4$ cm. Duljina kraće dijagonale trapeza iznosi:

A. $\sqrt{17}$ cm B. $\sqrt{21}$ cm C. $2\sqrt{5}$ cm D. $\sqrt{19}$ cm E. $3\sqrt{2}$

Nacrtajmo skicu:





1. Translatiramo stranicu d u vrh C -pa dobijemo trokut EBC iz kojeg izračunamo α i β

Prema kosinusovom teoremu:

$$b^2 = d^2 + (a-c)^2 - 2\cdot d\cdot(a-c)\cdot\cos a$$

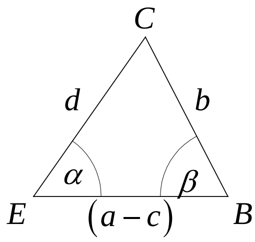
$$3^2 = 4^2 + 2^2 - 2\cdot 4\cdot 2\cdot\cos\alpha$$

$$9 - 16 - 4 = -16\cdot\cos\alpha$$

$$-16\cos\alpha = -11 \qquad / :(-16)$$

$$\cos\alpha = 0,6875 \qquad / \cos^{-1}$$

$$\alpha = 46^\circ 34' 03''$$



2.

$$\frac{b}{\sin\alpha} = \frac{d}{\sin\beta} \qquad / \cdot\sin\beta$$

$$\frac{b\cdot\sin\beta}{\sin\alpha} = d \qquad / \cdot\frac{\sin\alpha}{b}$$

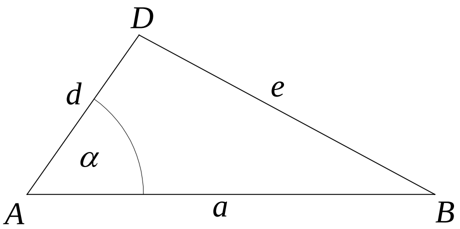
$$\sin\beta = \frac{d\cdot\sin\alpha}{b}$$

$$\sin\beta = \frac{4\cdot\sin 46^\circ 34' 03''}{3}$$

$$\sin\beta = \frac{4\cdot 0,726185}{3}$$

$$\sin\beta = 0,968246 \qquad / \sin^{-1}$$

$$\beta = 75^\circ 31' 21''$$



3. Pravilo kaže:
Nasuprot manjem kutu u trokutu nalazi se manja stranica.
Mi imamo dva trokuta ABD i ABC koji nas zanimaju tj. zanimaju nas njihove str. e i f kako smo izračunali da je $\alpha < \beta$ tada je i $e < f$ (prema gornjem pravilu).

$$e^2 = d^2 + a^2 - 2\cdot d\cdot a\cdot\cos\alpha$$

$$e^2 = 4^2 + 6^2 - 2\cdot 4\cdot 6\cdot\cos 46^\circ 34' 2''$$

$$e^2 = 16 + 36 - 48\cdot 0,687503$$

$$e^2 = 52 - 33,000146$$

$$e^2 = 19 \qquad / \sqrt{\ }$$

$$e = \sqrt{19}$$

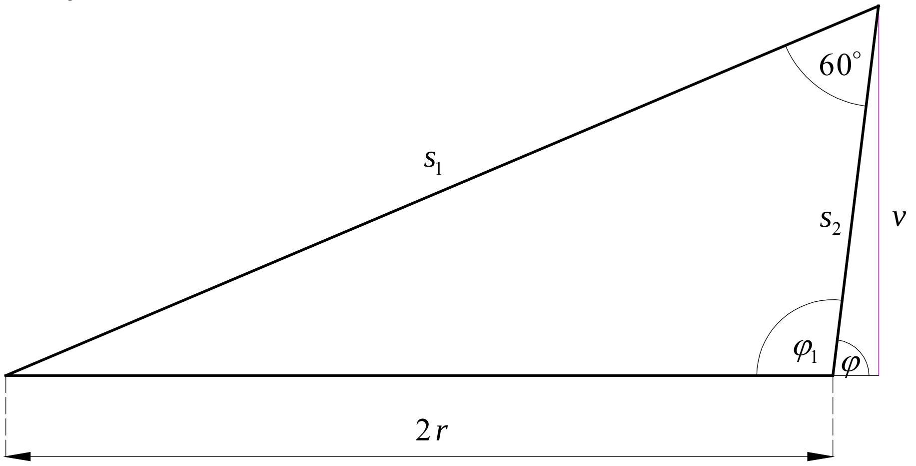
Za provjeru možete izračunati:

$$f^2 = a^2 + b^2 - 2\cdot a\cdot b\cdot\cos\beta$$

M-16. Kut pri vrhu karakterističnog presjeka kosog kružnog stošca iznosi 60°, duljina najdulje izvodnice iznosi 30, a najkraće 14. Volumen stošca iznosi:

A. $676\cdot\pi$ B. $105\sqrt{3}\cdot\pi$ C. $455\sqrt{3}\cdot\pi$ D. $315\sqrt{3}\cdot\pi$ E. $525\sqrt{2}\cdot\pi$

Nacrtajmo skicu:



1. Primjenimo kosinusov teorem i odredimo r:

$$(2r)^2 = s_1^2 + s_2^2 - 2\cdot s_1\cdot s_2\cdot\cos 60^\circ$$
$$(2r)^2 = 30^2 + 14^2 - 2\cdot 30\cdot 14\cdot\frac{1}{2}$$
$$(2r)^2 = 900 + 196 - 420$$
$$(2r)^2 = 676 \qquad / \sqrt{}$$
$$2r = 26$$
$$r = 13$$

2. Još jednom kosinusovim teoremom odredimo φ_1 :

$$s_1^2 = (2r)^2 + s_2^2 - 2\cdot(2r)\cdot s_2\cdot\cos\varphi_1$$
$$30^2 = 26^2 + 14^2 - 2\cdot 26\cdot 14\cdot\cos\varphi_1$$
$$900 = 676 + 196 - 728\cdot\cos\varphi_1$$
$$900 - 676 - 196 - 728\cdot\cos\varphi_1$$
$$28 = -728\cdot\cos\varphi_1$$
$$728\cdot\cos\varphi_1 = -28 \qquad / :728$$
$$\cos\varphi_1 = -0{,}0384615 \qquad / \cos^{-1}$$
$$\varphi_1 = 92^\circ 12' 15''$$

3. $\varphi_1 + \varphi = 180^\circ$

$$\varphi = 180^\circ - \varphi_1$$
$$\varphi = 180^\circ - 92^\circ 12' 15''$$
$$\varphi = 87^\circ 47' 45''$$

4. $\sin\varphi = \dfrac{v}{s_2} \qquad / \cdot s_2$

$$v = s_2\cdot\sin\varphi$$
$$v = 14\cdot\sin 87^\circ 47' 45''$$
$$v = 14\cdot 0{,}99926$$
$$v = 13{,}98964$$

5. $V = \dfrac{B\cdot v}{3}$

$$V = \frac{r^2\cdot\pi\cdot v}{3}$$
$$V = \frac{13^2\cdot\pi\cdot 13{,}98964}{3}$$
$$V = 788{,}083\cdot\pi = 455\cdot\sqrt{3}\cdot\pi$$

M-17. Zbroj rješenja jednadžbe $\frac{2\sin^2 x+1}{\sin x}=3$ u intervalu $(0,2\pi)$ iznosi:

A. $\frac{\pi}{2}$ B. $\frac{5\pi}{6}$ C. π D. $\frac{3\pi}{2}$ E. $\frac{4\pi}{3}$

$\frac{2\sin^2 x+1}{\sin x}=3$ $\rightarrow$ Uvjet: nazivnik mora biti različit od nule, dakle:

$\sin x \neq 0$

$\sin x \neq \sin k\pi$

$x \neq k\pi$

$\frac{2\sin^2 x+1}{\sin x}=3 \quad / \cdot \sin x$

$2\sin^2 x+1=3\sin x$

$2\sin^2 x-3\sin x+1=0$

Uvedemo: $\sin x = t$

$$t_{1,2}=\frac{-(-3)\pm\sqrt{(-3)^2-4\cdot 2\cdot 1}}{2\cdot 2}=\frac{3\pm\sqrt{9-8}}{4}=\frac{3\pm 1}{4}$$

$t_1=\frac{3+1}{4}=\frac{4}{4}=1$ $t_2=\frac{3-1}{4}=\frac{2}{4}=\frac{1}{2}$

$t_1=1$ $t_2=\frac{1}{2}$

Vratimo: $\mathrm{t}=\sin x$

$\sin x=1$ $\sin x=\frac{1}{2}$

$\sin x=\sin\left(\frac{\pi}{2}+2k\pi\right)$ $\sin x=\sin\left(\frac{\pi}{6}+2k\pi\right)$ $\sin x=\sin\left(\frac{5\pi}{6}+2k\pi\right)$

$x_1=\frac{\pi}{2}+2k\pi$ $x_2=\frac{\pi}{6}+2k\pi$ $x_3=\frac{5\pi}{6}+2k\pi$

Zadano je:

$x\in(0\ ,2\pi)$, takve x-ove dobijemo jedino za $k=0$, za $k=1,2,...$ x – ovi su izvan $(0\ ,2\pi)$

Za $k=0$

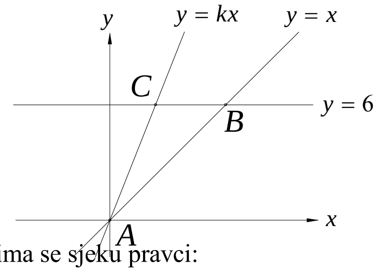
$x_1=\frac{\pi}{2}$ $x_2=\frac{\pi}{6}$ $x_3=\frac{5\pi}{6}$

Zbroj rješenja je: $x_1+x_2+x_3=\frac{\pi}{2}+\frac{\pi}{6}+\frac{5\pi}{6}=\frac{3\pi+\pi+5\pi}{6}=\frac{9\pi}{6}=\frac{3\pi}{6}$

M-18. Za koju vrijednost broja $k \geq 1$ površina trokuta što ga omeđuju pravci $y = x$, $y = kx$ i $y = 6$ iznosi 3 ?

A. $k = \frac{3}{2}$ B. $k = \frac{4}{3}$ C. $k = \frac{5}{4}$ D. $k = \frac{6}{5}$ E. $k = \frac{7}{6}$

Nacrtajmo skicu:



Odredimo točke u kojima se sijeku pravci:

$y = x$ i $y = kx$

supstitucijom $y = x$

dobijemo:

$$x = kx$$
$$x - kx = 0$$
$$x \cdot (1 - k) = 0 \quad / :(1-k)$$
$$x = 0$$
$$y = x$$
$$y = 0$$
$$A = (0, 0)$$

$y = x$ i $y = 6$

$$6 = x$$
$$x = 6$$
$$B = (,6,6)$$

$y = kx$ i $y = 6$

$$6 = kx$$
$$kx = 6 \quad / :k$$
$$x = \frac{6}{k}$$
$$C = \left(\frac{6}{k}, 6 \right)$$

$A = (0, 0)$ $B = (,6,6)$ $C = \left(\frac{6}{k}, 6 \right)$

$x_1 = 0$, $y_1 = 0$ $x_2 = 6$, $y_2 = 6$ $x_3 = \frac{6}{k}$, $y_3 = 6$

Zadana je površina trokuta kojeg zatvaraju ta tri pravca: $P = 3$

Formula kaže:

Površina trokuta zadanog sa tri točke

$$P = \frac{1}{2} \left| x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) \right|$$ uvrstimo sve poznato:

$$3 = \frac{1}{2} \cdot \left| 0 \cdot (6 - 6) + 6 \cdot (6 - 0) + \frac{6}{k} \cdot (0 - 6) \right| \quad / \cdot 2$$

$$6 = \left| 0 + 6 \cdot 6 + \frac{6}{k} \cdot (-6) \right|$$

$$6 = \left| 36 - \frac{36}{k} \right|$$

$$6 = 36 - \frac{36}{k}$$
$$\frac{36}{k} = 36 - 6$$
$$\frac{36}{k} = 30 \quad / \cdot k$$
$$36 = 30k \quad / :30$$
$$\frac{36}{30} = k$$
$$k = \frac{36}{30}$$
$$k = \frac{6}{5}$$

$$6 = -\left(36 - \frac{36}{k} \right)$$
$$6 = -36 + \frac{36}{k}$$
$$6 + 36 = \frac{36}{k}$$
$$42 = \frac{36}{k} \quad / \cdot k$$
$$42k = 36$$
$$k = \frac{36}{42}$$
$$k_2 = \frac{6}{7}$$

Kako je zadano $k \geq 1$ to $k_2 = \frac{6}{7}$ otpada, jedino rješenje je $k = \frac{6}{5}$

M-19. Na kružnicu $(x+1)^2+(y-1)^2=2$ povučene su tangente u točkama u kojima kružnice sijeku os *x*. Kut među tangentama iznosi:

A. 80° B. 85° C. 90° D. 95° E. 100°

1. Odredimo točke u kojima kružnica siječe $x-$os

Sve točke na $x-$osi imaju $y-$koordinatu nulta tj. $y=0$

$$(x+1)^2+(y-1)^2=2 \quad , \quad y=0$$
$$(x+1)^2+(0-1)^2=2$$
$$(x+1)^2+1=2$$
$$(x+1)^2=2-1$$
$$(x+1)^2=1 \quad / \sqrt{\ }$$
$$x+1=\pm 1$$

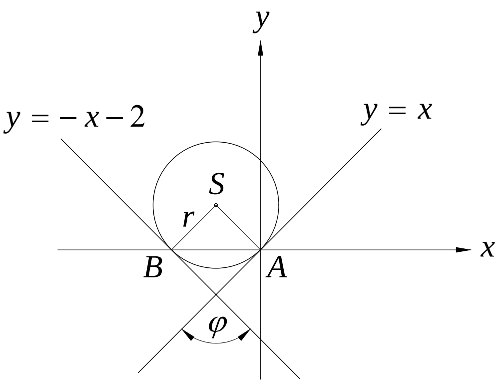
$x+1=1$ | $x+1=-1$

$x=1-1$ | $x=-1-1$

$x=0$ | $x=-2$

$y=0$

$A=(0,0)$ | $B=(-2,0)$



2. Sada u tim točkama odredimo jednadžbe tangenti na kružnicu:

$$(x+1)^2+(y-1)^2=2$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$p=-1 \quad q=1 \quad r^2=2$$

Jednadžba tangente u točki $T(x_1,y_1)$

$$(x_1-p)\cdot(x-p)+(y_1-q)\cdot(y-q)=r^2$$

$A=(0,0)$

$x_1=0 \quad y_1=0$

$$(x_1-p)\cdot(x-p)+(y_1-q)\cdot(y-q)=r^2$$
$$(0-(-1))\cdot(x-(-1))+(0-1)\cdot(y-1)=2$$
$$1\cdot(x+1)+(-1)\cdot(y-1)=2$$
$$x+1-y+1=2$$
$$x-y=2-1-1$$
$$x-y=0$$
$$-y=-x \quad / \ (-1)$$
$$y=x$$
$$k_1=1$$

$B=(-2,0)$

$x_1=-2 \quad y_1=0$

$$(x_1-p)\cdot(x-p)+(y_1-q)\cdot(y-q)=r^2$$
$$(-2-(-1))\cdot(x-(-1))+(0-1)\cdot(y-1)=2$$
$$(-2+1)\cdot(x+1)+(-1)\cdot(y-1)=2$$
$$-1\cdot(x+1)-y+1=2$$
$$-x-1-y+1=2$$
$$-y=x+2 \quad / \ (-1)$$
$$y=-x-2$$
$$\downarrow$$
$$k_2=-1$$

3. Kut tih tangenti odredimo preko formule:

$$\operatorname{tg}\varphi=\left|\frac{k_2-k_1}{1+k_1\cdot k_2}\right|=\left|\frac{-1-1}{1+1\cdot(-1)}\right|=\left|\frac{-2}{0}\right|=\infty$$

$$\operatorname{tg}\varphi=\infty \quad \Rightarrow \quad \varphi=90^\circ$$

M-20. Koordinate vrhova trokuta su $A(1,-2)$, $B(6,3)$, $C(-1,8)$. Udaljenost težišta trokuta od stranice AB iznosi:

A. $2\sqrt{2}$ B. $\sqrt{2}$ C. $6\sqrt{2}$ D. 2 E. $\sqrt{5}$

$A=(\underset{x_1}{1},\underset{y_1}{-2})$, $B=(\underset{x_2}{6},\underset{y_2}{3})$, $C=(\underset{x_3}{-1},\underset{y_3}{8})$

1. Odredimo koordinate težišta:

$$x_T=\frac{x_1+x_2+x_3}{3} \qquad y_T=\frac{y_1+y_2+y_3}{3}$$

$$x_T=\frac{1+6-1}{3}=\frac{6}{3} \qquad y_T=\frac{-2+3+8}{3}=\frac{9}{3}$$

$$x_T=2 \qquad y_T=3$$

$$T=(2,3)$$

2. Odredimo jednadžbu pravca AB

$A=(\underset{x_1}{1},\underset{y_1}{-2})$ $B=(\underset{x_2}{6},\underset{y_2}{3})$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}\cdot(x-x_1)$$

$$y-(-2)=\frac{3-(-2)}{6-1}\cdot(x-1)$$

$$y+2=\frac{5}{5}\cdot(x-1)$$

$$y+2=x-1$$

$$-x+y+2+1=0$$

$$-x+y+3=0 \qquad /\ (-1)$$

$$x-y-3=0$$

$\downarrow\ \downarrow\ \downarrow$

$A=1, B=-1, C=-3$, $T=(2,3)\Rightarrow x_1=2$, $y_1=3$

3. Udaljenost težišta trokuta od str. AB je:

$$d=\frac{|A\cdot x_1+B\cdot y_1+C|}{\sqrt{A^2+B^2}}$$

$$d=\frac{|1\cdot 2+(-1)\cdot 3-3|}{\sqrt{1^2+(-1)^2}}$$

$$d=\frac{|2-3-3|}{\sqrt{1+1}}=\frac{|-4|}{\sqrt{2}}$$

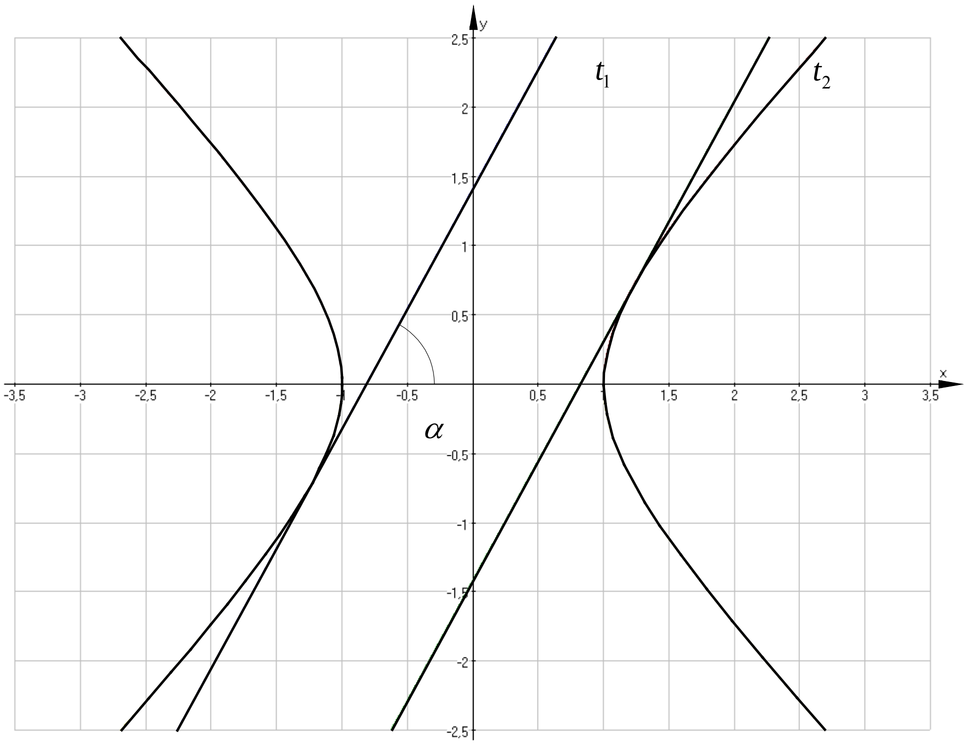
$$d=\frac{4}{\sqrt{2}}=\frac{4}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{4\sqrt{2}}{\sqrt{2^2}}=\frac{4\sqrt{2}}{2}$$

$$d=2\sqrt{2}$$

M-21. Pravac t_1 tangira lijevu, a njemu paralelni pravac t_2 desnu granu hiperbole $x^2 - y^2 = 1$.
Ako ti pravci sijeku os x pod kutom od 60°, onda je njihova međusobna udaljenost jednaka

A. $\frac{3}{2}$ B. $\sqrt{2}$ C. 2 D. $\sqrt{3}$ E. 3

1. Nacrtajmo sliku:



2. Pravci s x-osi zatvaraju kut od 60° $\Rightarrow$ $\alpha = 60^\circ$ kako znamo da je:

$$k = \text{tg}\,\alpha$$
$$k = \text{tg}\,60^\circ$$
$$k = \sqrt{3}$$

$k -$ je koeficjent smjera tangente (pravca)

$$\left.\begin{aligned} & x^2 - y^2 = 1 \\ & \frac{x^2}{1} - \frac{y^2}{1} = 1 \end{aligned}\right\} \text{Odredimo } a^2 \text{ i } b^2$$

$a^2 = 1$, $b^2 = 1$

3. Uvjet dodira pravca i hiperbole:

$$a^2k^2 - b^2 = l^2$$
$$1 \cdot \sqrt{3}^2 - 1 = l^2$$
$$3 - 1 = l^2$$
$$l^2 = 2 \qquad / \sqrt{\ }$$
$$l = \pm\sqrt{2}$$

4. Tražene tangente imaju jednadžbe:

$t_1 \ldots$ $y = kx + l_1$

$$y = \sqrt{3} \cdot x + \sqrt{2}$$
$$\sqrt{3} \cdot x - y + \sqrt{2} = 0$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$A = \sqrt{3}\ ,\ B = -1\ ,\ C_1 = \sqrt{2}$$

$t_2 \ldots$ $y = kx + l_2$

$$y = \sqrt{3} \cdot x + \sqrt{2}$$
$$\sqrt{3} \cdot x - y + \sqrt{2} = 0$$
$$\downarrow$$
$$C_2 = \sqrt{2}$$

5. Udaljenost paralelnih pravaca je:

$$\text{d} = \frac{|C_2 - C_1|}{\sqrt{A^2 + B^2}}$$

$$d = \frac{\left|-\sqrt{2} - \sqrt{2}\right|}{\sqrt{\sqrt{3}^2 + (-1)^2}} = \frac{\left|-2\sqrt{2}\right|}{\sqrt{3+1}} = \frac{2\sqrt{2}}{\sqrt{4}} = \frac{2\sqrt{2}}{2} = \sqrt{2} \qquad \Rightarrow \quad d = \sqrt{2}$$

M-22. Koordinate točaka jednako udaljenih od središta kružnica $x^2+y^2-2x+2y=0$ i $x^2+y^2+2x-2y=0$ zadovoljavaju jednadžbu:

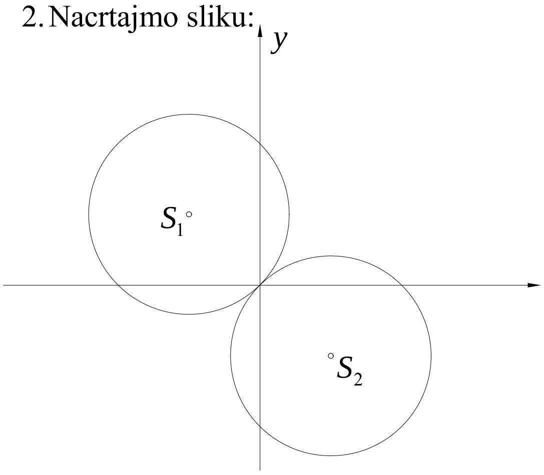
A. $x-y=1$ B. $x-y=0$ C. $x+y=1$ D. $x+y=0$ E. $y-x=1$

1. Odredimo koordinate središta tih kružnica nadopunjavanjem na potpuni kvadrat:

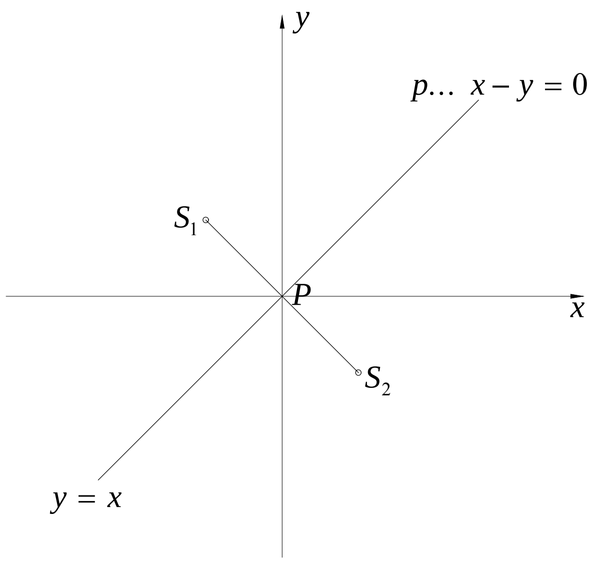
$$x^2+y^2-2x+2y=0$$
$$x^2-2x+1-1+y^2+2x+1-1=0$$
$$(x-1)^2-1+(y+1)^2-1=0$$
$$(x-1)^2+(y+1)^2=2$$
$$\downarrow \qquad \downarrow$$
$$-p=-1 \qquad -q=1$$
$$p=1 \qquad q=-1$$
$$S_1=(\underset{x_1}{1},\underset{y_1}{-1})$$

$$x^2+y^2+2x-2y=0$$
$$x^2+2x+1-1+y^2-2y+1-1=0$$
$$(x+1)^2-1+(y-1)^2-1=0$$
$$(x+1)^2+(y-1)^2=2$$
$$\downarrow \qquad \downarrow$$
$$-p=1 \qquad -q=-1$$
$$p=-1 \qquad q=1$$
$$S_2=(\underset{x_2}{-1},\underset{y_2}{1})$$

2. Nacrtajmo sliku:



3.
Tražimo točke koje su jednako udaljene od središta ovih kružnica...
Polovište dužine $\overline{S_1S_2}$ je sigurno jednako udaljeno od obadva središta, a i sve točke koje se nalaze na pravcu koji je okomit na pravac kroz S_1 i S_2 i koji prolazi kroz polovište dužine $\overline{S_1S_2}$...
↓



4. Odredim polovište dužine $\overline{S_1S_2}$

$$\left.\begin{aligned} x_p&=\frac{x_1+x_2}{2}=\frac{1-1}{2}=0\\ y_p&=\frac{y_1+y_2}{2}=\frac{-1+1}{2}=0\end{aligned}\right\} P=(0,0)$$

$$k_{S_1S_2}=\frac{y_2-y_1}{x_2-x_1}=\frac{1-(-1)}{-1-1}=\frac{2}{-2}=-1$$

$$k_p=-\frac{1}{k_{S_1S_2}}$$

$$k_p=-\frac{1}{-1}=1$$

5. $P=(0,0)$, $k_p=1$

$$p\ldots\ y-y_1=k_p\cdot(x-x_1)$$
$$y-0=1\cdot(x-0)$$
$$y=x$$

6. Jednadžba traženog pravca je

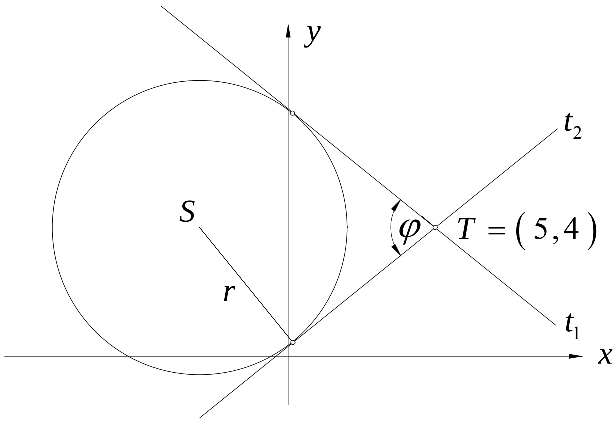
$$y=x$$
$$-x+y=0 \qquad /\cdot(-1)$$
$$x-y=0$$

M-23. Pod kojim se kutom iz točke $T(5,4)$ vidi kružnica $x^2+y^2+6x-8y=0$?

A. $77^\circ 22'$ B. 80° C. 60° D. 75° E. $82^\circ 38'$

1. Odredimo koordinate središta i polumjer kružnice...nadopunjavanjem na potpuni kvadrat...

$$x^2+y^2+6x-8y=0$$
$$x^2+6x+9-9+y^2-8y+16-16=0$$
$$(x+3)^2-9+(y-4)^2-16=0$$
$$(x+3)^2+(y-4)^2=25$$
$$\downarrow \qquad \downarrow \qquad \searrow$$
$$-p=3 \qquad -q=-4 \qquad r^2=25$$
$$p=-3 \qquad q=4 \qquad r=5$$
$$S=(-3,4)$$



2. Zadana je točka T iz koje se gleda kružnica, kordinate te točke moraju zadovoljavati jednadžbu tangente: $T=(5,4)$

$$y=kx+l$$
$$4=k\cdot 5+l \quad \leftarrow \quad x=5 \quad y=4$$
$$4-5k=l$$
$$l=4-5k$$

3. Uvjet dodira pravca i kružnice glasi:

$$(-k\cdot p+q-l)^2=r^2\cdot(1+k^2) \quad , \quad p=-3 \quad q=4 \quad r=5 \quad l=4-5k$$
$$(-k\cdot(-3)+4-(4-5k))^2=5^2\cdot(1+k^2)$$
$$(3k+4-4+5k)^2=25\cdot(1+k^2)$$
$$(8k)^2=25+25k^2$$
$$64k^2-25k^2=25$$
$$39k^2=25 \quad / :39$$
$$k^2=\frac{25}{39} \quad / \sqrt{\ } \Rightarrow \quad k=\pm\frac{5}{\sqrt{39}} \quad \Rightarrow k_1=-\frac{5}{\sqrt{39}}\, k_2=\frac{5}{\sqrt{39}}$$

4. Imamo koeficjente smjera obadvije tangente pa sada samo izračunamo kut koji zatvaraju te dvije tangente- to je traženi kut...

$$\operatorname{tg}\varphi=\left|\frac{k_2-k_1}{1+k_1\cdot k_2}\right|$$

$$\operatorname{tg}\varphi=\left|\frac{\frac{5}{\sqrt{39}}-\left(-\frac{5}{\sqrt{39}}\right)}{1+\left(-\frac{5}{\sqrt{39}}\right)\cdot\frac{5}{\sqrt{39}}}\right|=\left|\frac{\frac{5}{\sqrt{39}}+\frac{5}{\sqrt{39}}}{1-\frac{5^2}{\sqrt{39^2}}}\right|=\left|\frac{\frac{10}{\sqrt{39}}}{1-\frac{25}{39}}\right|=\left|\frac{\frac{10}{\sqrt{39}}}{\frac{39-25}{39}}\right|=\left|\frac{\frac{10}{\sqrt{39}}}{\frac{14}{39}}\right|=\left|\frac{390}{14\sqrt{39}}\right|$$

$$\operatorname{tg}\varphi=\frac{390}{14\sqrt{39}}=4{,}4607128 \quad / \operatorname{tg}^{-1}$$

$$\varphi=77^\circ 21' 52''=77^\circ 22'$$

M-24. Skup $A=\{(x,y)\ \in\ \mathrm{R}^2\ :\ x^2+4y^2-2x-16y+17=0\}$ je

A. elipsa B. hiperbola C. par pravaca D. parabola E. točka

$$x^2+4y^2-2x-16y+17=0$$
$$x^2-2x+4y^2-16y+17=0$$
$$x^2-2x+1-1+4\cdot\left(y^2-4y\right)+17=0$$
$$(x-1)^2-1+4\cdot\left(y^2-4y+4-4\right)+17=0$$
$$(x-1)^2-1+4\cdot\left[(y-2)^2-4\right]+17=0$$
$$(x-1)^2-1+4\cdot(y-2)^2-16+17=0$$
$$(x-1)^2+4\cdot(y-2)^2+17-1-16=0$$
$$(x-1)^2+4\cdot(y-2)^2=0$$