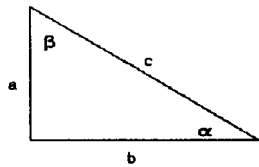


4. Trigonometrija pravokutnog trokuta

4.1. Definicije trigonometrijskih funkcija šiljastog kuta



$$\sin \alpha = \frac{a}{c}$$

$$\operatorname{tg} \alpha = \frac{a}{b}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\cos \alpha = \frac{b}{c}$$

$$\operatorname{ctg} \alpha = \frac{b}{a}$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\operatorname{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha}$$

1. , 1) $a = 4 \text{ cm}$
 $c = 9 \text{ cm}$
 $\sin \alpha = ?$ $\sin \beta = ?$
 $\cos \alpha = ?$ $\cos \beta = ?$
 $\operatorname{tg} \alpha = ?$ $\operatorname{tg} \beta = ?$
 $\operatorname{ctg} \alpha = ?$ $\operatorname{ctg} \beta = ?$

$$\sin \alpha = \frac{a}{c} = \frac{4}{9}$$

$$\sin \beta = \frac{b}{c} = \frac{\sqrt{65}}{9}$$

$$\cos \alpha = \frac{b}{c} = \frac{\sqrt{65}}{9}$$

$$\cos \beta = \frac{a}{c} = \frac{4}{9}$$

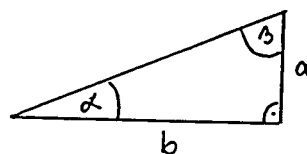
$$\operatorname{tg} \alpha = \frac{a}{b} = \frac{4}{\sqrt{65}}$$

$$\operatorname{tg} \beta = \frac{b}{a} = \frac{\sqrt{65}}{4}$$

$$\operatorname{ctg} \alpha = \frac{b}{a} = \frac{\sqrt{65}}{4}$$

$$\operatorname{ctg} \beta = \frac{a}{b} = \frac{4}{\sqrt{65}}$$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ b &= \sqrt{c^2 - a^2} \\ b &= \sqrt{9^2 - 4^2} \\ b &= \sqrt{81 - 16} \\ b &= \sqrt{65} \\ b &= 8,062 \end{aligned}$$



1.

$$2) \quad \begin{array}{l} a = 6 \text{ cm} \\ b = 9 \text{ cm} \\ \hline \alpha = ? \\ \beta = ? \end{array}$$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c &= \sqrt{a^2 + b^2} \\ c &= \sqrt{6^2 + 9^2} \\ c &= \sqrt{36 + 81} \\ c &= \sqrt{117} \\ c &\approx 10,82 \end{aligned}$$

$$\sin \alpha = \frac{a}{c} = \frac{6}{\sqrt{117}}$$

$$\cos \alpha = \frac{b}{c} = \frac{9}{\sqrt{117}}$$

$$\operatorname{tg} \alpha = \frac{a}{b} = \frac{6}{9} = \frac{2}{3}$$

$$\operatorname{ctg} \alpha = \frac{b}{a} = \frac{9}{6} = \frac{3}{2}$$

$$\sin \beta = \frac{b}{c} = \frac{9}{\sqrt{117}}$$

$$\cos \beta = \frac{a}{c} = \frac{6}{\sqrt{117}}$$

$$\operatorname{tg} \beta = \frac{b}{a} = \frac{9}{6} = \frac{3}{2}$$

$$\operatorname{ctg} \beta = \frac{a}{b} = \frac{6}{9} = \frac{2}{3}$$

$$3) \quad \begin{array}{l} b = 4 \text{ cm} \\ c = 8 \text{ cm} \\ \hline \end{array}$$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ a^2 &= c^2 - b^2 \\ a &= \sqrt{c^2 - b^2} \\ a &= \sqrt{8^2 - 4^2} \\ a &= \sqrt{64 - 16} \\ a &= \sqrt{48} \\ a &\approx 6,93 \end{aligned}$$

$$\sin \alpha = \frac{a}{c} = \frac{\sqrt{48}}{8}$$

$$\cos \alpha = \frac{b}{c} = \frac{4}{8} = \frac{1}{2}$$

$$\operatorname{tg} \alpha = \frac{a}{b} = \frac{\sqrt{48}}{4}$$

$$\operatorname{ctg} \alpha = \frac{b}{a} = \frac{4}{\sqrt{48}}$$

$$\sin \beta = \frac{b}{c} = \frac{4}{8} = \frac{1}{2}$$

$$\cos \beta = \frac{a}{c} = \frac{\sqrt{48}}{8}$$

$$\operatorname{tg} \beta = \frac{b}{a} = \frac{4}{\sqrt{48}}$$

$$\operatorname{ctg} \beta = \frac{a}{b} = \frac{\sqrt{48}}{4}$$

$$4) \quad \begin{array}{l} a = 15 \text{ cm} \\ b = 9 \text{ cm} \\ \hline \end{array}$$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c &= \sqrt{a^2 + b^2} \\ c &= \sqrt{15^2 + 9^2} \\ c &= \sqrt{225 + 81} \\ c &= \sqrt{306} \\ c &\approx 17,49 \end{aligned}$$

$$\sin \alpha = \frac{a}{c} = \frac{15}{\sqrt{306}}$$

$$\cos \alpha = \frac{b}{c} = \frac{9}{\sqrt{306}}$$

$$\operatorname{tg} \alpha = \frac{a}{b} = \frac{15}{9} = \frac{5}{3}$$

$$\operatorname{ctg} \alpha = \frac{b}{a} = \frac{9}{15} = \frac{3}{5}$$

$$\sin \beta = \frac{b}{c} = \frac{9}{\sqrt{306}}$$

$$\cos \beta = \frac{a}{c} = \frac{15}{\sqrt{306}}$$

$$\operatorname{tg} \beta = \frac{b}{a} = \frac{9}{15} = \frac{3}{5}$$

$$\operatorname{ctg} \beta = \frac{a}{b} = \frac{15}{9} = \frac{5}{3}$$

specijal-"BOS-ov" cjenik 2017.
20-60% POPUSTA NA ZBIRKE IZ TABLICE
PO DIJELOVIMA ili po POGLAVLJIMA
 ovi POPUSTI vrijede od 02.01.2017. do 21.03.2017.g.

Br.	Naziv ZBIRKE riješenih zadataka	Puna cijena	02.01.17.- 11.02.17. 30-60%	12.02.17.- 21.03.17. 15-40%
	SREDNJA ŠKOLA			
1.	Matematika-1- po Dakić-Elezović	740 kn	333kn	435 kn
1.A	ili samo I – polugodište (I ,II, III poglavlje)	335 kn	159 kn	220 kn
1.E	ili samo NEJEDNADŽBE ... 2.8.—4.4..	200 kn	99 kn	130 kn
1.B	II – polugodište (IV, V, VII, VIII poglavlje)	465 kn	199 kn	325 kn
9.	Testovi po Dakiću Mat-1- komplet rješenja - Zbirke zadataka s pismenih ispita – Mat-1	555 kn	315 kn	360 kn
9.A	ili samo I – polugodište (I - II grupa)	200 kn	111 kn	150 kn
9.B	II- polugodište (III, V, VI, VII grupa)	355 kn	199 kn	266 kn
11.	Matematika -2- - Dakić-Elezović			
11.A	ili samo I - polugodište (I- IV poglavlja)	450 kn	199 kn	250 kn
11.B	II - polugodište (V - VII poglavlja)	450 kn	229 kn	295 kn
	Matematika -2- Dakić-Elezović po dijelovima:			
11.C	KOMPLEKSNI BROJEVI	100 kn	60 kn	80 kn
11.D	KVADRATNA JEDNADŽBA	120 kn	80 kn	90 kn
11.E	POLINOMI 2. STUPNJA	120 kn	80 kn	90 kn
11.F	TRIGONOMETRIJA pravokutnog trokuta	140 kn	80 kn	90 kn
11.G	Eksponencijalne i logaritamske funkcije	150 kn	90 kn	100 kn
12.	Fizika-2- po Mikuličić, Varičak, Vernić (toplina i elektricitet)	240 kn	140 kn	155 kn
13.	Testovi po Dakiću Mat-2- (I-VI grupa) rješenja - Zbirke zadataka s pismenih ispita – Mat-2	540 kn	310 kn	351 kn
13.A	ili samo I – polugodište (I-IV grupa)	340 kn	199 kn	238 kn
13.B	II – polugodište (V-VI grupa)	200 kn	150 kn	170 kn
	ili svako poglavlje – za sebe	155 kn	50 kn	79 kn
14.	Matematika-3 - Dakić-Elezović			
14.A	ili samo I - polugodište (trigonometrija)	410 kn	199 kn	270 kn
14.B	II-polugodište (vektori i analitička geometrija)	440 kn	239 kn	289 kn

	Matematika-3 Dakić-Elezović - po DIJELOVIMA			
	1. polugodište			
14.A-1	Trigonometrija 1. dio – svi zadaci od. 1.1.- 3.3.	210 kn	99 kn	140 kn
14.A-2	Trigonometrija 2. dio – svi zadaci od. 3.3.- 6.5.	210 kn	99 kn	140 kn
	2. polugodište			
14.C	VEKTORI – svi zadaci od. 7.1. – do 7.7.	120 kn	80 kn	90 kn

Kod narudžbe matematičkih zbirki riješenih zadataka bitn

Za sve

Ostale zbirke potpuno riješenih zadataka koje vam trebaju kontaktirajte nas mailo mili putem telefona ...

ZA SVE INFORMACIJE I NARUDŽBE MOŽETE SE OBRATITI SVAKI DAN OD 09⁰⁰ – 20⁰⁰ ili mailom od 0-do-24

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na tel/fax 01-4578-431 , tel 01-4579-130 mob: 098-237-534

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OVO JE SAMO JEDAN MANJI DIO OGLEDNIH PRIMJERA POTPUNO RIJEŠENIH ZADATAKA PO ŠKOLSKOJ ZBIRCI DAKIĆ-ELEZOVIĆ

**U NASTAVKU
IMATE
20 stranica
sa detaljno riješenim zadacima
uz većinu tih zadataka
imamo i prateće video instrukcije i upute
pogled na neke video
instrukcije
[imate ovdje link >>>>](#)**

2.

Pravougaonik trokut

$$\begin{array}{l} a = 5 \\ b = 12 \end{array}$$

$$\begin{array}{l} \sin \alpha = ? \\ \cos \alpha = ? \\ \operatorname{tg} \alpha = ? \\ \operatorname{ctg} \alpha = ? \end{array}$$

hipotenuza c:

$$\begin{array}{l} c^2 = a^2 + b^2 \\ c^2 = 5^2 + 12^2 \\ c^2 = 25 + 144 \\ c = \sqrt{169} \\ c = 13 \end{array}$$

$$\sin \alpha = \frac{a}{c}$$

$$\sin \alpha = \frac{5}{13}$$

$$\cos \alpha = \frac{b}{c}$$

$$\cos \alpha = \frac{12}{13}$$

$$\operatorname{tg} \alpha = \frac{a}{b}$$

$$\operatorname{tg} \alpha = \frac{5}{12}$$

$$\operatorname{ctg} \alpha = \frac{b}{a}$$

$$\operatorname{ctg} \alpha = \frac{12}{5}$$

3.

$$a = 7 \text{ cm}$$

$$c = 25 \text{ cm}$$

$$\operatorname{tg} \alpha \text{ i } \operatorname{ctg} \alpha = ?$$

$$1. \quad b^2 = c^2 - a^2$$

$$b^2 = 25^2 - 7^2$$

$$b^2 = 576 / \sqrt{\quad}$$

$$b = 24 \text{ cm}$$

$$2. \quad \operatorname{tg} \alpha = \frac{a}{b}$$

$$\operatorname{tg} \alpha = \frac{7}{24}$$

$$4. \quad \operatorname{tg} \beta = \frac{b}{a}$$

$$\operatorname{tg} \beta = \frac{24}{7}$$

$$3. \quad \operatorname{ctg} \alpha = \frac{b}{a}$$

$$\operatorname{ctg} \alpha = \frac{24}{7}$$

$$5. \quad \operatorname{ctg} \beta = \frac{a}{b}$$

$$\operatorname{ctg} \beta = \frac{7}{24}$$

8. Pravokutan trokut :

$$1) \quad \begin{array}{l} a = 4 \text{ cm} \\ \sin \alpha = \frac{2}{3} \\ \hline c = ? \end{array}$$

$$\begin{aligned} \sin \alpha &= \frac{a}{c} \\ \frac{2}{3} &= \frac{4}{c} \quad / \cdot 3c \\ 2c &= 4 \cdot 3 / : 2 \\ c &= \frac{12}{2} \\ c &= 6 \text{ cm} \end{aligned}$$

$$2) \quad \begin{array}{l} a = 4 \text{ cm} \\ \operatorname{tg} \alpha = \frac{2}{3} \\ \hline b = ? \end{array}$$

$$\begin{aligned} \operatorname{tg} \alpha &= \frac{a}{b} \\ \frac{2}{3} &= \frac{4}{b} \quad / \cdot 3b \\ 2b &= 4 \cdot 3 \\ 2b &= 12 / : 2 \\ b &= 6 \text{ cm} \end{aligned}$$

$$3) \quad \begin{array}{l} c = 4 \text{ cm} \\ \sin \alpha = \frac{2}{3} \\ \hline a = ? \end{array}$$

$$\begin{aligned} \sin \alpha &= \frac{a}{c} \\ \frac{2}{3} &= \frac{a}{4} \quad / \cdot 12 \\ 4 \cdot 2 &= 3 \cdot a \\ 3a &= 8 / : 3 \\ a &= \frac{8}{3} \text{ cm} \end{aligned}$$

$$4) \quad \begin{array}{l} c = 4 \text{ cm} \\ \cos \alpha = \frac{2}{3} \\ \hline b = ? \end{array}$$

$$\begin{aligned} \cos \alpha &= \frac{b}{c} \\ \frac{2}{3} &= \frac{b}{4} \quad / \cdot 12 \\ 4 \cdot 2 &= 3 \cdot b \\ 3b &= 8 / : 3 \\ b &= \frac{8}{3} \text{ cm} \end{aligned}$$

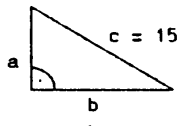
9.

HIPOTENUZA JE NAJDULJA STRANICA PRAVOKUTNOG TROKUTA

zadano

$$c = 15$$

$$\sin \alpha = \frac{2}{3}$$



$$\text{kako je } \sin \alpha = \frac{a}{c} \quad | \quad \sin \alpha = \frac{2}{3}$$

$$\text{tada je } \frac{a}{c} = \frac{2}{3} \quad \text{uvrstimo } c = 15$$

$$\frac{a}{15} = \frac{2}{3} \quad / \cdot 15$$

$$a = \frac{2 \cdot 15}{3}$$

$$a = 10$$

dalje po pitagorinom teoremu

$$c^2 = a^2 + b^2 \quad \text{imamo} \quad b^2 = c^2 - a^2$$

$$b^2 = 15^2 - 10^2$$

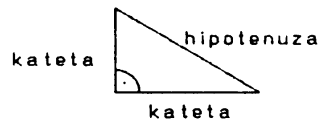
$$b^2 = 225 - 100$$

$$b^2 = 125 \quad / \sqrt{\quad}$$

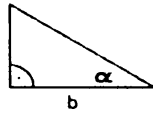
$$b = \sqrt{125} = \sqrt{25 \cdot 5}$$

$$b = 5\sqrt{5}$$

10.



$b = 15 \rightarrow$ jedna kateta sada je svjedno da li je a ili b



\rightarrow kut priležeći kateti b je α

$$\sin \alpha = \frac{4}{5}$$

$$\sin \alpha = \frac{\text{kateta nasuprot kuta}}{\text{hipotenuza}} = \frac{a}{c}$$

pa iz toga dobijemo

$$\sin \alpha = \frac{4}{5} \quad \text{i} \quad \sin \alpha = \frac{a}{c}$$

$$\frac{a}{c} = \frac{4}{5} \quad / \cdot c$$

$$a = \frac{4}{5} c$$

$$c^2 = a^2 + b^2$$

$$c^2 = \left(\frac{4}{5}c\right)^2 + 15^2$$

$$\frac{9}{25}c^2 = 225 \quad / \cdot \frac{25}{9}$$

$$c^2 = \frac{16}{25}c^2 + 225$$

$$c^2 = \frac{225 \cdot 25}{9} = \frac{9 \cdot 25 \cdot 25}{9}$$

$$c^2 - \frac{16}{25}c^2 = 225$$

$$c^2 = 625 \quad / \sqrt{\quad}$$

$$\left(1 - \frac{16}{25}\right) \cdot c^2 = 225$$

$$c = 25 \quad a = \frac{4}{5}c$$

$$a = \frac{4}{5} \cdot 25 = 20$$

11.

$$\cos \alpha = \frac{3}{4} \quad b = 9$$

$$\cos \alpha = \frac{b}{c} \quad \text{i} \quad \cos \alpha = \frac{3}{4}$$

$$\frac{b}{c} = \frac{3}{4}$$

$$\frac{9}{c} = \frac{3}{4} \quad / \cdot 4 \cdot c$$

$$36 = 3 \cdot c \quad / : 3$$

$$12 = c \rightarrow c = 12 \rightarrow$$

$$\rightarrow c^2 = a^2 + b^2$$

$$\uparrow a^2 = c^2 - b^2$$

$$a^2 = 12^2 - 9^2$$

$$\uparrow a^2 = 144 - 81$$

$$a^2 = 63 \quad / \sqrt{\quad}$$

$$a = \sqrt{63} = \sqrt{9 \cdot 7}$$

$$a = 3\sqrt{7}$$

15.

Kada je zadani kut šiljat to znači da je α u intervalu od 0° do 90° .
vrijednosti sa $\sin \alpha$ i $\cos \alpha$ u tom intervalu su od 0 do 1 pa pišemo :

$$0 < \sin \alpha < 1 \quad \text{i} \quad 0 < \cos \alpha < 1$$

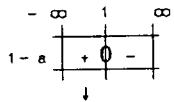
$$1.) \sin \alpha = \frac{1}{1-a} \quad 0 < \sin \alpha < 1$$

$$0 < \frac{1}{1-a} < 1$$

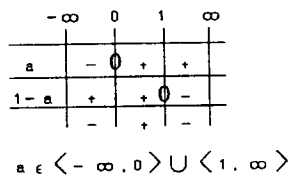
$$2.) \cos \alpha = \frac{2a}{a^2+1} \quad 0 < \cos \alpha < 1$$

$$0 < \frac{2a}{a^2+1} < 1$$

<p>I</p> $0 < \frac{1}{1-a}$ $\frac{1}{1-a} > 0$ $1-a = 0$ $-a = -1 \quad / \cdot (-1)$ $a = 1$	<p>II</p> $\frac{1}{1-a} < 1$ $\frac{1}{1-a} - 1 < 0$ $\frac{1-1 \cdot (1-a)}{1-a} < 0$ $\frac{-1+1+a}{1-a} < 0$ $\frac{a}{1-a} < 0$ $a = 0 \quad 1-a = 0$ $-a = -1 \quad / \cdot (-1)$ $a = 1$
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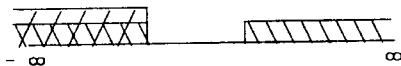


$$a \in \langle -\infty, 1 \rangle$$



$$a \in \langle -\infty, 0 \rangle \cup \langle 1, \infty \rangle$$

ukupno rješenje je presjek I i II



$$a \in \langle -\infty, 0 \rangle$$

rješenje možemo pisati i $a < 1$

<p>I</p> $0 < \frac{2a}{a^2+1}$	<p>II</p> $\frac{2a}{a^2+1} < 1$
---------------------------------	----------------------------------

kako je nazivnik a^2+1 pozitivan za bilo koji $a \in \mathbb{R}$ to obadviije nejednadžbe možemo množiti sa a^2+1

$$0 < \frac{2a}{a^2+1} \quad / \cdot (a^2+1) \quad \frac{2a}{a^2+1} < 1 \quad / \cdot (a^2+1)$$

$$0 < 2a$$

$$2a > 0 \quad / : 2$$

$$a > 0$$

$$2a < a^2+1$$

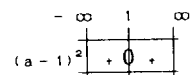
$$0 < a^2-2a+1$$

$$(a-1)^2 > 0$$

$$\downarrow$$

$$\text{nul točka } a-1=0$$

$$a=1$$



$$(a-1)^2 > 0$$

$$\downarrow$$

$$a > 0$$

$$a \in \mathbb{R} \setminus \{1\}$$

ukupno rješenje je presjek I i II



$$a \in \langle 0, \infty \rangle \setminus \{1\}$$

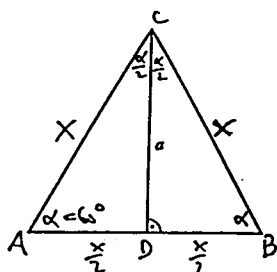
ili

$$a > 0 \quad \text{i} \quad a \neq 1$$

4.2. Vrijednosti trigonometrijskih funkcija kuteva od 30° , 45° , 60°

1.

1.)



STRANICU OZNAČIMO SA (x) TAKO DA NE RIJEŠIMO...
IZ PRAMOKUTNOG TRIKUTA ADC IMAMO:

$$\sin \alpha = \frac{a}{x} \quad / \cdot x$$

$$x \cdot \sin \alpha = a \quad / : \sin \alpha$$

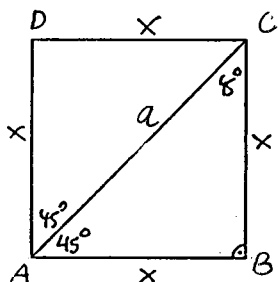
$$x = \frac{a}{\sin \alpha}$$

$$x = \frac{a}{\sin 60^\circ} = \frac{a}{\frac{\sqrt{3}}{2}} = \frac{2a}{\sqrt{3}} = \frac{2a}{\sqrt{3}}$$

$$x = \frac{2a}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2a\sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{2\sqrt{3}}{3} a$$

$$x = \frac{2\sqrt{3}}{3} a$$

2.)



STRANICU OZNAČIMO SA x

IZ PRAMOKUTNOG TRIKUTA ABC IMAMO
PO PITAGORINOM TEOREMU

$$a^2 = x^2 + x^2$$

$$a^2 = 2x^2 \quad / : 2$$

$$\frac{a^2}{2} = x^2 \Rightarrow$$

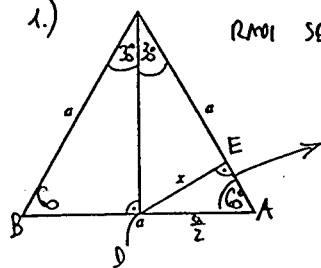
$$x^2 = \frac{a^2}{2} \quad / \sqrt{\quad}$$

$$x = \frac{\sqrt{a^2}}{\sqrt{2}} = \frac{a}{\sqrt{2} \cdot \sqrt{2}}$$

$$x = \frac{a\sqrt{2}}{2} = \frac{\sqrt{2}}{2} a$$

2.

1.)



IMO SE JEDNAKOSTRANICAN TRIKUTU KA JE $\alpha = 60^\circ$

IZ PRAMOKUTNOG TRIKUTA AED IMAMO

$$\sin 60^\circ = \frac{x}{\frac{a}{2}} \quad \rightarrow \quad \frac{\sqrt{3}}{2} x = \frac{x}{\frac{a}{2}} = \frac{2x}{a}$$

$$\sin 60^\circ = \frac{2x}{a}$$

$$\frac{\sqrt{3}}{2} = \frac{2x}{a} \quad / \cdot \frac{a}{2}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{a}{2} = x$$

$$x = \frac{a\sqrt{3}}{4} = \frac{\sqrt{3}}{4} a$$

4.3. Računanje vrijednosti trigonometrijskih funkcija

4.3.

4.3.

6. 1) $a = 5, c = 13$

$$\begin{aligned} \textcircled{1} \quad b^2 &= c^2 - a^2 \\ b^2 &= 13^2 - 5^2 \\ b^2 &= 144 / \sqrt{} \\ \underline{b} &= \underline{12} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad \sin \alpha &= \frac{a}{c} \\ \sin \alpha &= \frac{5}{13} \end{aligned}$$

$$\begin{aligned} \sin \alpha &= 0,38461538 / \sin^{-1} \\ \alpha &= \underline{22^\circ 37' 12''} \end{aligned}$$

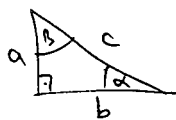
ovo računamo računalom (DIGITRON)



$$\begin{aligned} \textcircled{3} \quad \cos \alpha &= \frac{b}{c} \\ \cos \alpha &= \frac{12}{13} \\ \cos \alpha &= 0,9230769 / \cos^{-1} \\ \alpha &= \underline{22^\circ 37' 12''} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad \sin \beta &= \frac{b}{c} \\ \sin \beta &= \frac{12}{13} \\ \sin \beta &= 0,9230769 / \sin^{-1} \\ \beta &= \underline{67^\circ 22' 48''} \end{aligned}$$

2) $b = 8, c = 15$



$$\begin{aligned} \textcircled{1} \quad \sin \alpha &= \frac{a}{c} \\ \sin \alpha &= \frac{\sqrt{161}}{15} \\ \sin \alpha &= 0,815905 / \sin^{-1} \\ \alpha &= \underline{57^\circ 46' 9''} \end{aligned}$$

$$\begin{aligned} a^2 &= c^2 - b^2 \\ a^2 &= 15^2 - 8^2 = 225 - 64 \\ a^2 &= 161 / \sqrt{} \\ \underline{a} &= \underline{\sqrt{161}} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad \sin \beta &= \frac{b}{c} \\ \sin \beta &= \frac{8}{15} \\ \sin \beta &= 0,533333 / \sin^{-1} \\ \beta &= \underline{32^\circ 13' 51''} \end{aligned}$$

3.) $a = 20$
 $b = 21$

$$\tan \alpha = \frac{a}{b}$$

$$\tan \alpha = \frac{20}{21}$$

$$\tan \alpha = 0,9523809 / \tan^{-1}$$

$$\alpha = \underline{43^\circ 36' 10''}$$

U pravokutnom trouglu

$$\text{uvijek je: } \alpha + \beta = 90^\circ$$

$$\text{pa je } \beta = 90^\circ - \alpha$$

$$\beta = 89^\circ 59' 60'' - 43^\circ 36' 10''$$

$$\beta = \underline{46^\circ 23' 50''}$$

$$|K| \textcircled{1} c^2 = a^2 + b^2$$

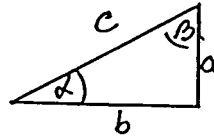
$$\textcircled{1} \quad \cos \beta = \frac{a}{c} \quad |K| \quad \sin \beta = \frac{b}{c}$$

$$\beta = \underline{46^\circ 23' 50''}$$

* U ovakvom tipu zadataka
ima više načina rješavanja
također rješavanja u svakom zadatku 1, 2, 3
primamo samo jedan od ponuđenih načina
Svi načini rješavanja su
dobri ako onaj točno rješete...

7. zadatak

Pravokutan trokut



$$\alpha + \beta = 90^\circ$$

$$\beta = 90^\circ - \alpha$$

$$\begin{array}{l} 1) \quad a = 3 \text{ cm} \\ \quad \quad b = 4 \text{ cm} \\ \hline c = ? \\ \alpha = ? \\ \beta = ? \end{array}$$

$$\text{tg } \alpha = \frac{a}{b}$$

$$\text{tg } \alpha = \frac{3}{4}$$

$$\text{tg } \alpha = 0,75$$

$$\alpha = \text{tg}^{-1} 0,75$$

$$\alpha = 36,8699^\circ$$

$$\alpha = 36^\circ 52' 12''$$

$$\beta = 90^\circ - \alpha$$

$$\beta = 90^\circ - 36^\circ 52' 12''$$

$$\beta = 53,1301^\circ$$

$$\beta = 53^\circ 7' 48''$$

$$\sin \alpha = \frac{a}{c} \quad | \cdot c$$

$$\sin \alpha \cdot c = a$$

$$c = \frac{a}{\sin \alpha} = \frac{3}{\sin 36^\circ 52' 12''}$$

$$c = \frac{3}{0,6} = 5 \text{ cm}$$

$$\begin{array}{l} 2) \quad a = 5 \text{ cm} \\ \quad \quad c = 7 \text{ cm} \\ \hline b = ? \\ \alpha = ? \\ \beta = ? \end{array}$$

$$\sin \alpha = \frac{a}{c}$$

$$\sin \alpha = \frac{5}{7} = 0,71429$$

$$\alpha = \sin^{-1} 0,71429$$

$$\alpha = 45,5847^\circ$$

$$\alpha = 45^\circ 35' 5''$$

$$\text{tg } \alpha = \frac{a}{b}$$

$$b = \frac{a}{\text{tg } \alpha} = \frac{5}{\text{tg } 45^\circ 35' 5''}$$

$$b = \frac{5}{1,0206}$$

$$b = 4,899 \text{ cm}$$

$$\beta = 90^\circ - \alpha$$

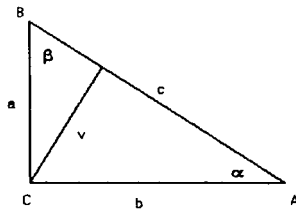
$$\beta = 90^\circ - 45^\circ 35' 5''$$

$$\beta = 44,4153^\circ$$

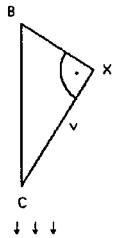
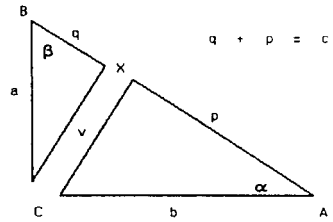
$$\beta = 44^\circ 24' 55''$$

18. zadano $v = 20,4 \text{ cm}$ i $\alpha = 32^\circ 24'$

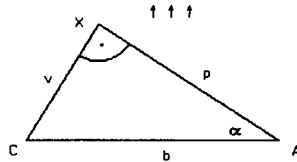
1.)



zadana je visina i jedan siljasti kut . . .
Visina je okomica iz vrha C na stranicu c
Kada bi prerezali taj trokut po visini doбили
bismo dva pravokutna trokuta → → →



uzmimo sada prvi trokut AXC → to je pravokutan trokut sa hipotenuzom b



$$\operatorname{tg} \alpha = \frac{v}{p} \quad / \cdot p$$

$$p \cdot \operatorname{tg} \alpha = v \quad / : \operatorname{tg} \alpha$$

$$p = \frac{v}{\operatorname{tg} \alpha}$$

$$p = \frac{20,4}{\operatorname{tg} 32^\circ 40'} = \frac{20,4}{0,64116734} = 31,8169667$$

$$p = 31,82 \text{ cm}$$

nakon toga uzmemo drugi trokut CXB

$$\alpha + \beta = 90^\circ$$

$$\beta = 90^\circ - \alpha$$

$$\beta = 89^\circ 60' - 32^\circ 40'$$

$$\beta = 57^\circ 20'$$

$$\operatorname{tg} \beta = \frac{v}{q} \quad / \cdot q$$

$$q \cdot \operatorname{tg} \beta = v \quad / : \operatorname{tg} \beta$$

$$q = \frac{v}{\operatorname{tg} \beta}$$

$$q = \frac{20,4}{\operatorname{tg} 57^\circ 20'} = \frac{20,4}{1,5596552}$$

$$q = 13,079814$$

$$q = 13,08 \text{ cm}$$

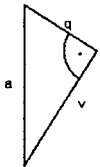
$$p = 31,82 \text{ cm}$$

$$q = 13,08 \text{ cm}$$

$$c = p + q$$

$$c = 31,82 + 13,08$$

$$c = 44,9 \text{ cm}$$



$$a^2 = v^2 + q^2$$

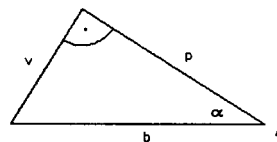
$$a^2 = 20,4^2 + 13,08^2$$

$$a^2 = 416,16 + 171,0864$$

$$a^2 = 587,2464 \quad / \sqrt{\quad}$$

$$a = 24,23316735$$

$$a = 24,23 \text{ cm}$$



$$b^2 = v^2 + p^2$$

$$b^2 = 20,4^2 + 31,82^2$$

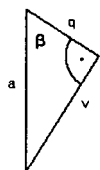
$$b^2 = 416,16 + 1012,5124$$

$$b^2 = 1428,6724 \quad / \sqrt{\quad}$$

$$b = 37,797783$$

$$b = 37,80 \text{ cm}$$

OVAJ zadatak mogli smo riješiti i na drugi način tako da prvo računamo a i b stranice pa tek onda c stranu . . .



$$\alpha + \beta = 90$$

$$\beta = 57^\circ 20'$$

$$\sin \beta = \frac{v}{a}$$

$$a = \frac{v}{\sin \beta} = \frac{20,4}{\sin 57^\circ 20'} = \frac{20,4}{0,84182494}$$

$$a = 24,23 \text{ cm}$$

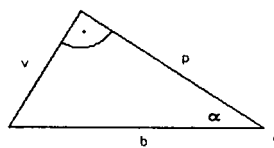
pa sada računamo c stranicu preko

$c^2 = a^2 + b^2$ bez računanja p i q . . . obadva načina su do
ovaj drugi je nešto kraći . . .

$$\operatorname{tg} \alpha = \frac{v}{p} \quad / \cdot p$$

$$p = \frac{v}{\operatorname{tg} \alpha}$$

isto kao i gore . . .



2.) zadano $v = 5,26 \text{ cm}$ i $\beta = 65^\circ 30'$ → tehnika je ista kao u 1.) prvo izračunamo α

$$\alpha = 90^\circ - \beta$$

$$\alpha = 89^\circ 60' - 65^\circ 30'$$

$$\alpha = 24^\circ 30'$$

$$q = \frac{v}{\operatorname{tg} \beta}$$

$$q = \frac{5,26}{\operatorname{tg} 65^\circ 30'} = \frac{5,26}{2,19429973}$$

$$q = 2,397120$$

$$q = 2,40 \text{ cm}$$

$$p = \frac{v}{\operatorname{tg} \alpha}$$

$$p = \frac{5,26}{\operatorname{tg} 24^\circ 30'} = \frac{5,26}{0,455726255}$$

$$p = 11,542 \text{ cm}$$

$$c = p + q$$

$$c = 11,542 + 2,40$$

$$c = 13,942$$

$$c = 13,94 \text{ cm}$$

$$a^2 = v^2 + q^2$$

$$a^2 = 5,26^2 + 2,4^2$$

$$a^2 = 27,6676 + 5,76$$

$$a^2 = 33,4276 \quad / \sqrt{\quad}$$

$$a = 5,78 \text{ cm}$$

$$b^2 = c^2 - a^2$$

$$b^2 = 13,94^2 - 5,78^2$$

$$b^2 = 194,3236 - 33,4084$$

$$b^2 = 160,9152 \quad / \sqrt{\quad}$$

$$b = 12,69 \text{ cm}$$

19. pravokutan trokut

$$P = 22 \text{ cm}^2$$

$$\angle = 56^\circ 40'$$

$$a = ?$$

$$b = ?$$

$$c = ?$$

$$P = \frac{a \cdot b}{2} / \cdot 2$$

$$2P = a \cdot b$$

$$\cos \alpha = \frac{b}{c}$$

$$b = c \cdot \cos \alpha$$

$$b = c \cdot \cos 56^\circ 40'$$

$$b = 0,5495 \cdot c$$

$$\sin \alpha = \frac{a}{c}$$

$$a = c \cdot \sin \alpha$$

$$a = c \cdot \sin 56^\circ 40'$$

$$a = c \cdot 0,83549$$

$$a = 0,83549 \cdot c$$

$$2P = a \cdot b$$

$$2 \cdot 22 = 0,83549 \cdot c \cdot 0,5495 \cdot c$$

$$44 = 0,4591 c^2 / : 0,4591$$

$$c^2 = \frac{44}{0,4591}$$

$$c^2 = 95,839686 / \sqrt{\quad}$$

$$c = 9,78977 \text{ cm}$$

$$c = 9,8 \text{ cm}$$

$$b = 0,5495 \cdot c$$

$$b = 0,5495 \cdot 9,8 \text{ cm}$$

$$b = 5,3857 \text{ cm}$$

$$b = 5,38 \text{ cm}$$

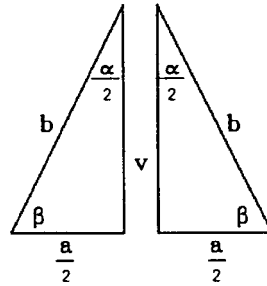
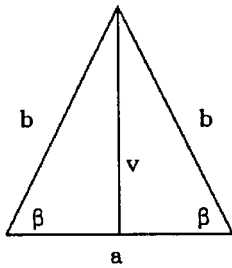
$$a = 0,83549 \cdot c$$

$$a = 0,83549 \cdot 9,8 \text{ cm}$$

$$a = 8,18 \text{ cm}$$

4.5. Primjene u planimetriji

1.



RASTAVIMO OVAJ JEDNAKOKRAČAN TROKUT PO
VISINI NA DVA PRAVOKUTNA TROKUTA

$$\sin \beta = \frac{v}{b} \qquad \cos \beta = \frac{a}{2 \cdot b}$$

$$\operatorname{tg} \beta = \frac{2 \cdot v}{a} \qquad \operatorname{ctg} \beta = \frac{a}{2 \cdot v}$$

a) $a = 6,5 \text{ cm}$
 $b = 11 \text{ cm}$

$$\cos \beta = \frac{a}{2 \cdot b}$$

$$\cos \beta = \frac{6,5}{2 \cdot 11} = \frac{6,5}{22}$$

$$\cos \beta = 0,295445 \quad / \cos^{-1}$$

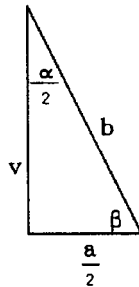
$$\beta = 72^{\circ} 48' 55''$$

$$\beta = 72^{\circ} 49'$$

$$\frac{\alpha}{2} = 90^{\circ} - 72^{\circ} 49'$$

$$\frac{\alpha}{2} = 89^{\circ} 60' - 72^{\circ} 49'$$

$$\frac{\alpha}{2} = 17^{\circ} 11' \quad / \cdot 2 \rightarrow \alpha = 34^{\circ} 22'$$



b) $a = 22,7 \text{ cm}$
 $b = 15,2 \text{ cm}$

$$\cos \beta = \frac{a}{2 \cdot b}$$

$$\cos \beta = \frac{22,7}{2 \cdot 15,2} = \frac{22,7}{30,4}$$

$$\cos \beta = 0,74671052 \quad / \cos^{-1}$$

$$\beta = 41^{\circ} 41' 38''$$

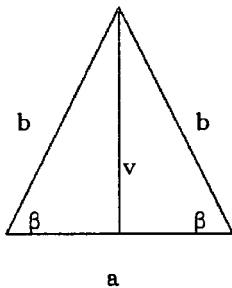
$$\beta = 41^{\circ} 42'$$

$$\frac{\alpha}{2} = 89^{\circ} 60' - 41^{\circ} 42'$$

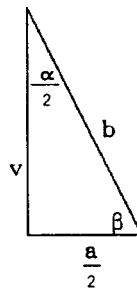
$$\frac{\alpha}{2} = 48^{\circ} 18' \quad / \cdot 2$$

$$\alpha = 96^{\circ} 36'$$

2.



← iz trokuta
izvadimo trokut →



$$\sin \frac{\alpha}{2} = \frac{a/2}{b} = \frac{a}{2 \cdot b}$$

$$\sin \frac{\alpha}{2} = \frac{a}{2 \cdot b}$$

$$\cos \frac{\alpha}{2} = \frac{v}{b}$$

1) $\alpha = 140^{\circ} \rightarrow \frac{\alpha}{2} = 70^{\circ}$
 $a = 20 \text{ cm}$

$$\sin \frac{\alpha}{2} = \frac{a}{2 \cdot b} \quad / \cdot b$$

$$b \cdot \sin \frac{\alpha}{2} = \frac{a}{2} \quad / : \sin \frac{\alpha}{2}$$

$$b = \frac{a}{2 \cdot \sin \frac{\alpha}{2}}$$

$$b = \frac{20}{2 \cdot \sin 70^{\circ}} = \frac{20}{2 \cdot 0,939693} = 10,64178$$

$$b = 10,64 \text{ cm}$$

$$\beta = 90^{\circ} - \frac{\alpha}{2} = 90^{\circ} - 70^{\circ}$$

$$\beta = 20^{\circ}$$

2) $\alpha = 55^{\circ} \rightarrow \frac{\alpha}{2} = 27^{\circ} 30'$
 $a = 8,5 \text{ cm}$

$$b = \frac{a}{2 \cdot \sin \frac{\alpha}{2}}$$

$$b = \frac{8,5}{2 \cdot \sin 27^{\circ} 30'} = \frac{8,5}{2 \cdot 0,4617486}$$

$$b = \frac{8,5}{0,923497} = 9,2041$$

$$b = 9,2 \text{ cm}$$

$$\beta = 90^{\circ} - \frac{\alpha}{2}$$

$$\beta = 89^{\circ} 60' - 27^{\circ} 30'$$

$$\beta = 62^{\circ} 30'$$

3. 1)

$b = 45 \text{ cm}$
 $\beta = 12^\circ$

$\cos \beta = \frac{a}{2 \cdot b} \quad / \cdot 2 \cdot b$

$2 \cdot b \cdot \cos \beta = a$

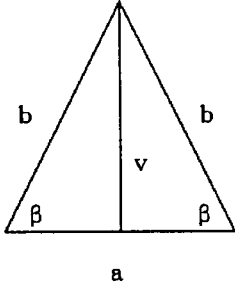
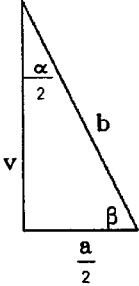
$a = 2 \cdot b \cdot \cos \beta$

$a = 2 \cdot 45 \cdot \cos 12^\circ$

$a = 90 \cdot 0,9781476$

$a = 88,03328$

$a = 88,03 \text{ cm}$

$\alpha + 2 \cdot \beta = 180^\circ$

$\alpha = 180^\circ - 2 \cdot \beta$

$\alpha = 180^\circ - 2 \cdot 12^\circ = 180^\circ - 24^\circ$

$\alpha = 156^\circ$

2) $b = 5,2 \text{ cm}$
 $\beta = 67^\circ 20'$

$a = 2 \cdot b \cdot \cos \beta$

$a = 2 \cdot 5,2 \cdot \cos 67^\circ 20'$

$a = 10,4 \cdot 0,3853693$

$a = 4,00784$

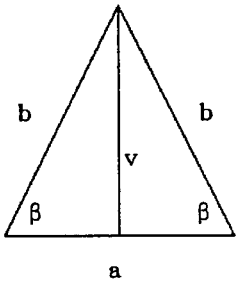
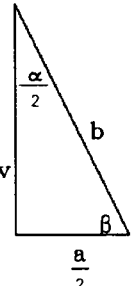
$a = 4,01 \text{ cm}$

$\alpha = 180^\circ - 2 \cdot \beta$

$\alpha = 180^\circ - 2 \cdot 67^\circ 20' = 179^\circ 60' - 134^\circ 40'$

$\alpha = 45^\circ 20'$

4. 1)

zaano α i v : računamo po \rightarrow

$\alpha = 101^\circ \rightarrow \frac{\alpha}{2} = 50^\circ 30'$

$v = 15 \text{ cm}$

$a = 2 \cdot v \cdot \lg \frac{\alpha}{2}$

$a = 2 \cdot 15 \cdot \lg 50^\circ 30'$

$a = 30 \cdot 1,213097$

$a = 36,3929$

$a = 36,39 \text{ cm}$

$b = \frac{v}{\cos \frac{\alpha}{2}}$

$b = \frac{15}{\cos 50^\circ 30'} = \frac{15}{0,636078} = 23,582$

$b = 23,58 \text{ cm}$

$\lg \frac{\alpha}{2} = \frac{a}{v}$

$\lg \frac{\alpha}{2} = \frac{a}{2 \cdot v} \quad / \cdot 2 \cdot v$

$2 \cdot v \cdot \lg \frac{\alpha}{2} = a$

$a = 2 \cdot v \cdot \lg \frac{\alpha}{2}$

$\sin \frac{\alpha}{2} = \frac{a}{2 \cdot b}$

$\cos \frac{\alpha}{2} = \frac{v}{b} \quad / \cdot b \rightarrow b \cdot \cos \frac{\alpha}{2} = v \quad / : \cos \frac{\alpha}{2}$

$b = \frac{v}{\cos \frac{\alpha}{2}}$

2) $\alpha = 33^\circ \rightarrow \frac{\alpha}{2} = 16^\circ 30'$

$v = 112 \text{ cm}$

$a = 2 \cdot v \cdot \lg \frac{\alpha}{2}$

$a = 2 \cdot 112 \cdot \lg 16^\circ 30'$

$a = 224 \cdot 0,2962135 = 66,35182$

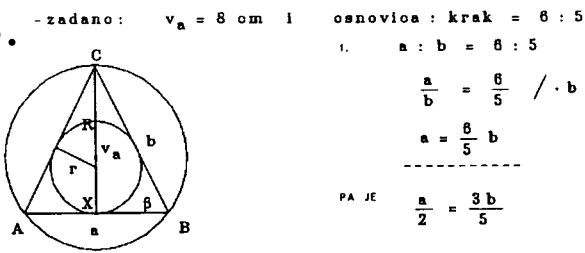
$a = 66,35 \text{ cm}$

$b = \frac{v}{\cos \frac{\alpha}{2}}$

$b = \frac{112}{\cos 16^\circ 30'} = \frac{112}{0,9588197} = 116,810278$

$b = 116,81 \text{ cm}$

18.



1. $a : b = 6 : 5$

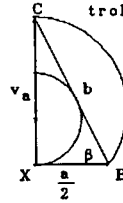
$$\frac{a}{b} = \frac{6}{5} \quad / \cdot b$$

$$a = \frac{6}{5} b$$

PA JE

$$\frac{a}{2} = \frac{3b}{5}$$

2. - iz jednakokrakog trokuta izdvojimo pravokutni trokut XBC i pomoću njega dodemo do kuta β ...



$$\cos \beta = \frac{a/2}{b}$$

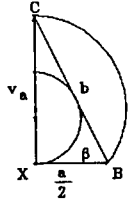
$$\cos \beta = \frac{3b}{5b} = \frac{3}{5}$$

$$\cos \beta = \frac{3}{5}$$

$$\cos \beta = 0,6 \quad / \cos^{-1}$$

$$\beta = 53^\circ 7' 48''$$

3. $\sin \beta = \frac{v_a}{b} \quad / \cdot \frac{b}{\sin \beta}$



$$b = \frac{v_a}{\sin \beta}$$

$$b = \frac{8}{\sin 53^\circ 7' 48''} = \frac{8}{0,79999893} = 10$$

$$b = 10 \text{ cm}$$

4. I SADA IZ ZADATKA 5.82. IMAMO FORMULU :

$$R = \frac{b}{2 \cdot \sin \beta}$$

$$R = \frac{10}{2 \cdot \sin 53^\circ 7' 48''} = \frac{5}{0,79999893} = 6,250836$$

$$R = 6,25 \text{ cm}$$

5. $a = \frac{6}{5} b = \frac{6}{5} \cdot 10 = 6 \cdot 2$

$$a = 12 \text{ cm}$$

6. I SADA IZ ZADATKA 5.82. IMAMO FORMULU :

$$r = \frac{a \cdot \operatorname{tg} \frac{\beta}{2}}{2}$$

$$r = \frac{12 \cdot \operatorname{tg} 26^\circ 33' 54''}{2} = 6 \cdot 0,499998883$$

$$r = 2,99999330$$

$$r = 3 \text{ cm}$$

19.

- zadano $O = 2 \text{ dm}$, $P = \frac{6}{25} \text{ dm}^2$

$$O = 2 \cdot (a + b) \quad P = a \cdot b$$

$$2 = 2 \cdot (a + b) \quad / : 2 \quad \frac{6}{25} = a \cdot b$$

$$1 = a + b$$

$$1 - a = b \quad / \cdot \frac{6}{25} = a \cdot (1 - a) \quad / \cdot 25$$

$$6 = 25a \cdot (1 - a)$$

$$6 = 25a - 25a^2$$

$$25a^2 - 25a + 6 = 0$$

$$a_{1,2} = \frac{-(-25) \pm \sqrt{(-25)^2 - 4 \cdot 25 \cdot 6}}{2 \cdot 25} = \frac{25 \pm \sqrt{25}}{50} = \frac{25 \pm 5}{50}$$

$$a_1 = \frac{25 + 5}{50} = \frac{30}{50} = \frac{3}{5}$$

$$a_2 = \frac{25 - 5}{50} = \frac{20}{50} = \frac{2}{5}$$

$$a_1 = \frac{3}{5}$$

$$a_2 = \frac{2}{5}$$

$$b = 1 - a$$

$$b_1 = 1 - \frac{3}{5} = \frac{5-3}{5} = \frac{2}{5}$$

$$b_2 = 1 - \frac{2}{5} = \frac{3}{5}$$

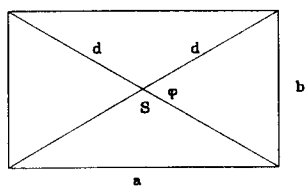
$$b_1 = \frac{2}{5}$$

$$b_2 = \frac{3}{5}$$

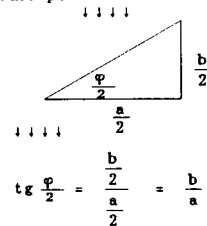
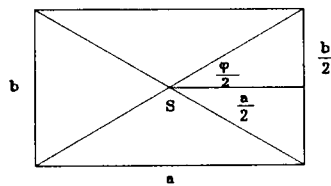
- $a = \frac{3}{5}$ i $b = \frac{2}{5}$

----> uzmimo samo a_1 i b_1 , jer za a_2 i b_2 radi se o istom pravokutniku...

- nacrtajmo taj pravokutnik označimo stranice i kut...



- iz sjecišta dijagonala spustimo visinu na b... dobili smo pravokutni trokut pa kut φ dobijemo:



$$\operatorname{tg} \frac{\varphi}{2} = \frac{b/2}{a/2} = \frac{b}{a}$$

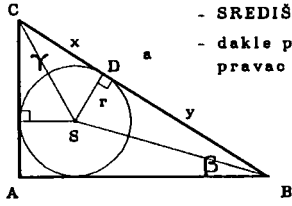
$$\operatorname{tg} \frac{\varphi}{2} = \frac{b}{a} \quad , \quad \operatorname{tg} \frac{\varphi}{2} = \frac{2}{3} = \frac{2}{3} \quad , \quad \operatorname{tg} \frac{\varphi}{2} = 0,66666 \quad / \operatorname{tg}^{-1} \quad , \quad \frac{\varphi}{2} = 33^\circ 41' 24'' \quad / \cdot 2$$

$$\varphi = 67^\circ 22' 48''$$

ovo je samo jedna manji dio detaljno riješenih zadataka ...kompletno sva rješenja mogu se nabaviti u štampanoj varijanti

- za detalje kontaktirajte nas na telefon – 01-4578-431 ili 098-237-534

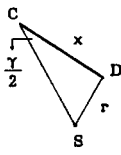
36. zadano : $\beta = 50^\circ$, $\gamma = 74^\circ$, polumjer upisane kruznice $r = 25$ cm , $a = ?$
 nacrtajmo sliku :



- SREDIŠTE TROKUTU UPISANE KRUZNICE JE SJECIŠTE SIMETRALA KUTEVA -
 - dakle pravac kroz točke C i S dijeli kut γ na dva jednaka djela , isto tako pravac kroz točke B i S dijeli kut β na dva jednaka djela . . .

- iz ΔABC izdvojimo dva pravokutna Δ

- ΔCSD



$$\text{tg } \frac{\gamma}{2} = \frac{r}{x}$$

$$\text{tg } \frac{74^\circ}{2} = \frac{25}{x} \quad / \cdot x$$

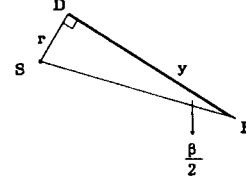
$$x \cdot \text{tg } 37^\circ = 25$$

$$x \cdot 0,75355 = 25 \quad / : 0,75355$$

$$x = 33,17629885$$

$$x = 33,18 \text{ cm}$$

- ΔBSD



$$\text{tg } \frac{\beta}{2} = \frac{r}{y}$$

$$\text{tg } \frac{50^\circ}{2} = \frac{25}{y} \quad / \cdot y$$

$$y \cdot \text{tg } 25^\circ = 25$$

$$y \cdot 0,46631 = 25 \quad / : 0,46631$$

$$y = 53,6124$$

$$y = 53,61 \text{ cm}$$

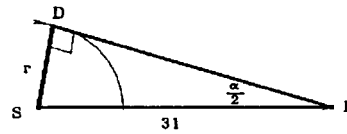
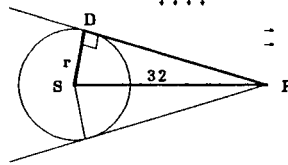
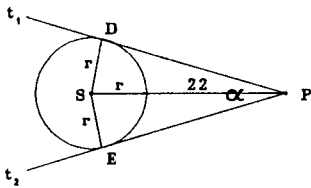
$$a = x + y$$

$$a = 33,18 + 53,61$$

$$a = 86,79 \text{ cm}$$

U RJEŠENJIMA ZBIRKE FORMULA JE OK. ALI UMJESTO α TREBA PISATI β I RAČUN IM NIJE DOBAR PROVJERITE SAMI UZMITE LOG. TABLICE I OČITAJTE $\text{ctg } 37^\circ$ I $\text{ctg } 25^\circ$ I UVRSTITE IH U NJIUVU FORMULU I DOBITI ĆETE MOJ O . . .

37. nacrtajmo sliku : prvo nacrtamo kružnicu zatim polumjer produžimo za 22 cm jer je udaljenost od središta S do točke P 31 cm kako je $r = 9$ to je udaljenost točke P od kružnice = 22 cm , dalje iz točke P povučemo tangente t na kružnicu , točke dodira su D i E , u tim točkama r je okomit na tangente . . . uočimo pravokutni ΔPSD



imamo :

$$\sin \frac{\alpha}{2} = \frac{r}{31}$$

$$\sin \frac{\alpha}{2} = \frac{9}{31}$$

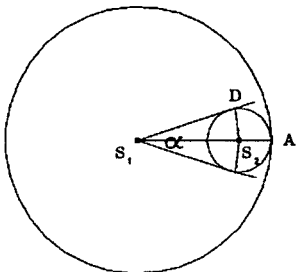
$$\sin \frac{\alpha}{2} = 0,29032 \quad / \sin^{-1}$$

$$\frac{\alpha}{2} = 16^\circ 52' 36'' \quad / \cdot 2$$

$$\alpha = 33^\circ 45' 12''$$

- zašto je r okomit na tangentu u točki D ?
 - jer je pravac kroz točke S i D (normala) okomit na tangentu a polumjer r nalazi se na tom pravcu . . .

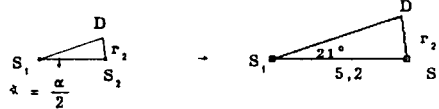
38. nacrtajmo sliku : - kružnice se diraju iznutra $d(S_1, S_2) = r_1 - r_2$



$$r_1 - r_2 = 5,2 \text{ cm}$$

$$\alpha = 42^\circ$$

- treba uočiti pravokutni $\Delta S_1 S_2 D$
 - izdvojimo ga



$$\text{tg } \frac{\alpha}{2} = \frac{r_2}{r_1 - r_2}$$

$$\sin 21^\circ = \frac{r_2}{5,2} \quad / \cdot 5,2$$

$$2. r_1 - r_2 = 5,2 \text{ om}$$

$$r_1 - 1,86 = 5,2$$

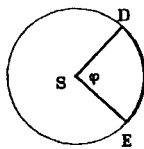
$$r_1 = 5,2 + 1,86$$

$$r_1 = 7,06 \text{ om}$$

$$5,2 \cdot \sin 21^\circ = r_2$$

$$r_2 = 5,2 \cdot 0,35837 = 1,863524$$

$$r_2 = 1,86 \text{ cm}$$

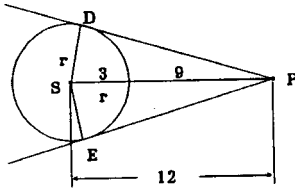


$l(\varphi)$ - duljina kružnog luka $l(\varphi) = \frac{r \cdot \pi}{180^\circ} \cdot \varphi$

- duljina kružnog luka je udaljenost točke D do E kada se ide po kružnici normalno sa onog djela "polukruga" kojeg određuje kut φ . . .

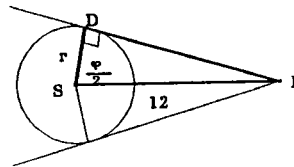
39. zadano $r = 3$ cm , $d(SP) = 12$ cm , traži se duljina kružnog luka . . .

- slika:

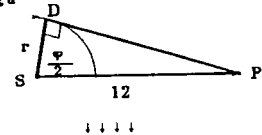


sl.1.

- na sl.1. uoči pravokutni ΔDSP



- izdvojimo ga



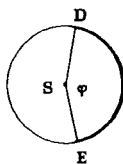
$$\cos \frac{\varphi}{2} = \frac{r}{12}$$

$$\cos \frac{\varphi}{2} = \frac{3}{12}$$

$$\cos \frac{\varphi}{2} = 0,25 \quad / \cos^{-1}$$

$$\frac{\varphi}{2} = 75^\circ 31' 21'' \quad / \cdot 2$$

$$\varphi = 151^\circ 2' 42''$$



$$l(\varphi) = \frac{r \cdot \pi}{180^\circ} \cdot \varphi$$

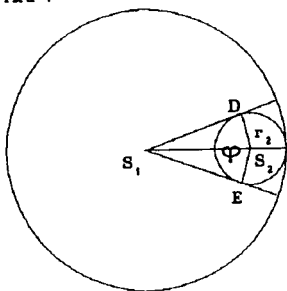
$$l = \frac{3 \cdot 3,14}{180^\circ} \cdot 151^\circ 2' 42''$$

$$l = \frac{9,42}{180^\circ} \cdot 151,045^\circ = 7,904888$$

$$l = 7,9 \text{ cm}$$

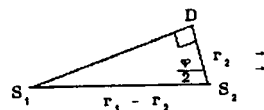
40. $r_2 = 3$ cm , $r_1 = 7$ cm

- slika :



$$d(S_1, S_2) = r_1 - r_2$$

- izdvojimo $\Delta S_1 S_2 D$



$$\cos \frac{\varphi}{2} = \frac{r_2}{r_1 - r_2}$$

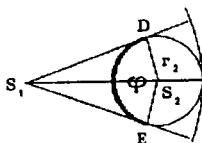
$$\cos \frac{\varphi}{2} = \frac{3}{7 - 3} = \frac{3}{4}$$

$$\cos \frac{\varphi}{2} = 0,75 \quad / \cos^{-1}$$

$$\frac{\varphi}{2} = 41^\circ 24' 35'' \quad / \cdot 2$$

$$\varphi = 82^\circ 49' 10''$$

- duljina luka manje kružnice koja se vidi iz S_1 je :



$$l(\varphi) = \frac{r \cdot \pi}{180^\circ} \cdot \varphi$$

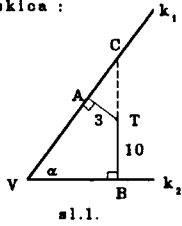
$$l = \frac{3 \cdot 3,14}{180^\circ} \cdot 82^\circ 49' 10'' = \frac{9,42}{180^\circ} \cdot 82,8194444^\circ = 4,33421759$$

$$l = 4,33 \text{ cm}$$

61. OVDJE SE POJAVLJUJE JEDAN PROBLEM RJEŠENJA ZBIRKE RADE KAO DA JE ZADANA UDALJENOST OD JEDNOG KRAKA 10 I DRUGOG 2 CM . DOK U ZBIRCI KOJU JA IMAM PIŠE DA JE UDALJENOST OD KRAKA 10 I 3 CM , JA ĆU RADITI PO ZADANOM ZADATKU . . .

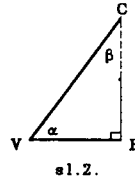
- zadano : točka T je udaljena od krakova kuta $\alpha = 75^\circ$ za 3 cm i 10 cm . . .
- traži se udaljenost od vrha V kuta α do točke T . . .

- skica :



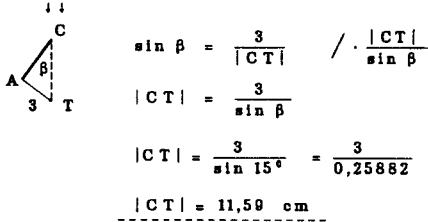
- najbliža udaljenost točke od kraka je njena okomita projekcija krak (u točke A i B)
- vrh kuta α označimo sa V
- produžimo pravac kroz točke B i T do kraka k_1 , sjecište označimo sa C

1. - iz pravokutnog ΔVBC dodemo do β



$$\begin{aligned} \beta &= 90^\circ - \alpha \\ \beta &= 90^\circ - 75^\circ \\ \beta &= 15^\circ \end{aligned}$$

2. - iz sl.1. izdvojimo pravokutni ΔCAT

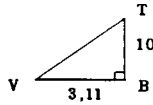
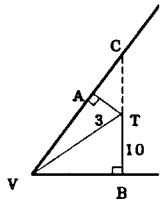


$$\begin{aligned} \sin \beta &= \frac{3}{|CT|} \quad / \cdot \frac{|CT|}{\sin \beta} \\ |CT| &= \frac{3}{\sin \beta} \\ |CT| &= \frac{3}{\sin 15^\circ} = \frac{3}{0,25882} \\ |CT| &= 11,59 \text{ cm} \end{aligned}$$

3. - iz sl.2. - imamo :

$$\begin{aligned} \text{tg } \beta &= \frac{|VB|}{|BC|} \quad / \cdot |VB| \\ |VB| &= |BC| \cdot \text{tg } \beta \\ |VB| &= 11,59 \cdot \text{tg } 15^\circ = 11,59 \cdot 0,26795 = 3,1055405 \\ |VB| &= 3,11 \text{ cm} \end{aligned}$$

4. - i sada tražimo udaljenost vrha V od točke T
- spojimo točke V i T i iz slike izdvojimo pravokutni ΔVBT

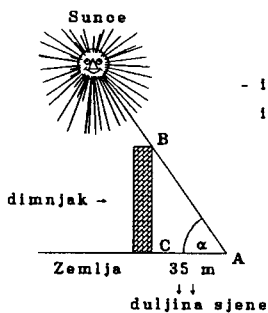


- po Pitagorinom teoremu imamo

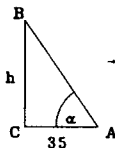
$$\begin{aligned} |VT|^2 &= |VB|^2 + |BT|^2 \\ |VT|^2 &= 3,11^2 + 10^2 = 9,6721 + 100 \\ |VT|^2 &= 109,6721 \quad / \sqrt{\quad} \\ |VT| &= 10,4724 \end{aligned}$$

62. - zadano : duljina sjene = 35 m , i kut pod kojim sunčeve zrake padaju na zemlju $\alpha = 55^\circ$, traži se visina dimnjaka . . .

- slika :



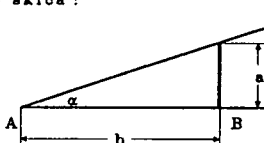
- iz ove moje prekrasne slike izdvojimo pravokutni ΔABC



$$\begin{aligned} \text{tg } \alpha &= \frac{h}{35} \quad / \cdot 35 \\ h &= 35 \cdot \text{tg } \alpha = 35 \cdot \text{tg } 55^\circ = 35 \cdot 1,428148 = 49,98518 \\ h &= 50 \text{ m} \quad \text{- tražena visina dimnjaka} \end{aligned}$$

63. - zadan je uspon ceste od 12%

- skica :



- a = visinska razlika
- b = horizontalni pomak

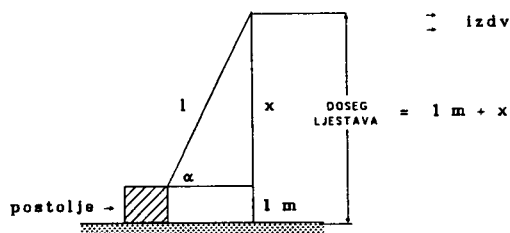
$$\text{uspon ceste u } \% = \frac{\text{visinska razlika}}{\text{horizontalni pomak}}$$

$$12 \% = \frac{a}{b} \quad , \text{ kako je } \text{tg } \alpha = \frac{a}{b}$$

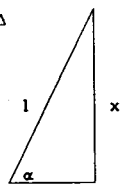
$$\begin{aligned} \text{tg } \alpha &= 12 \% \\ \text{tg } \alpha &= \frac{12}{100} = 0,12 \\ \text{tg } \alpha &= 0,12 \quad / \text{tg}^{-1} \\ \alpha &= 6^\circ 50' 34'' \end{aligned}$$

- α - je kut pod kojim se cesta uspinje u odnosu na horizontali . . .

64. zadano : dužina ljestava $l = 30 \text{ m}$, postolja visine $h = 1 \text{ m}$, najveći nagib 70°
 skica :



izdvojimo pravokutni Δ



$$\sin \alpha = \frac{x}{l} \quad / \cdot l$$

$$x = l \cdot \sin \alpha$$

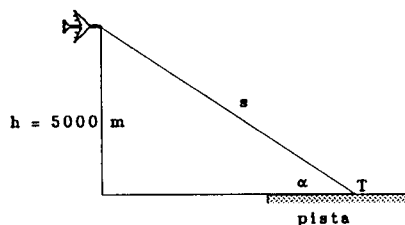
$$x = 30 \cdot \sin 70^\circ$$

$$x = 30 \cdot 0,93969 = 28,1886$$

$$x = 28,19 \text{ m}$$

- pa je doseg ljestava = visina postolja + $x = 1 + 28,19 = 29,19 \text{ m}$

65. zadano :
 zrakoplov je na visini od 5000 m - $h = 5000 \text{ m}$, stalni kut α pod kojim se spušta je $15,5^\circ$ - $\alpha = 15,5^\circ$
 - koliko je zrakoplov udaljen od točke dodira sa slijetnom pistom - dakle traži se put koji treba preletjeti do točke T u kojoj će dodirnuti pistu ... iz slike je vidljivo da je hipotenuza pravokutnog trokuta taj traženi put.
 - slika:



$$\sin \alpha = \frac{h}{s} \quad / \cdot \frac{s}{\sin \alpha}$$

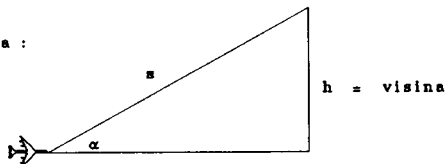
$$s = \frac{h}{\sin \alpha}$$

$$s = \frac{5000}{\sin 15,5^\circ} = \frac{5000}{0,26724} = 18709,77399 \text{ m}$$

$$s = 18709,77 \text{ m ili } 18,709 \text{ km}$$

66. zadano : kut uspinjanja $\alpha = 40^\circ$
 brzina zrakoplova $v = 300 \text{ km/h}$, i vrijeme uspinjanja $t = 10 \text{ sek}$, traži se visina na koju se popne nakon 10 sek !
 $v = 300 \cdot \frac{1000}{3600} = 83,3333 \text{ m/sek}$

- slika :



1. izračunamo pređeni put s za $t = 10 \text{ sek}$

$$s = v \cdot t$$

$$s = 83,3333 \cdot 10 = 8333,33 \text{ m}$$

$$s = 833,333 \text{ m}$$

2. - visina na koju se popne zrakoplov je h ... :

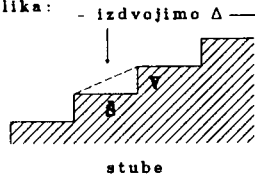
$$\sin \alpha = \frac{h}{s} \quad / \cdot s$$

$$h = s \cdot \sin \alpha = 833,333 \cdot \sin 40^\circ = 833,333 \cdot 0,642788 = 535,65645$$

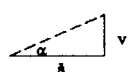
$$h = 535,656 \text{ m} = 536 \text{ m}$$

67. zadano : visina stuba - $v = 14 \text{ cm}$, i širina stuba $\delta = 30 \text{ cm}$, kut α uspinjanja stubišta je ?

- slika :



izdvojimo Δ



$$\text{tg } \alpha = \frac{v}{\delta}$$

$$\text{tg } \alpha = \frac{14}{30}$$

$$\text{tg } \alpha = 0,46667 \quad / \text{tg}^{-1}$$

$$\alpha = 25^\circ 1' 2''$$

$$\alpha = 25^\circ$$

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PO DIJELOVIMA ili po POGLAVLJIMA
 ovi POPUSTI vrijede od 02.01.2017. do 21.03.2017.g.

Br.	Naziv ZBIRKE riješenih zadataka	Puna cijena	02.01.17.- 11.02.17. 30-60%	12.02.17.- 21.03.17. 15-40%
	SREDNJA ŠKOLA			
1.	Matematika-1- po Dakić-Elezović	740 kn	333 kn	435 kn
1.A	ili samo I – polugodište (I ,II, III poglavlje)	335 kn	159 kn	220 kn
1.E	ili samo NEJEDNADŽBE ... 2.8.—4.4..	200 kn	99 kn	130 kn
1.B	II – polugodište (IV, V, VII, VIII poglavlje)	465 kn	199 kn	325 kn
	ili po djelovima (poglavljima)			
1. C	Potencije – algebarski izrazi i Algebarski razlomci Svi zadaci od 1.1. pa do 2.7. (320 stranica ...)	200 kn	99 kn	120 kn
1.D	Jednadžbe , nejednadžbe i nejednadžbe s apsolutnim vrijednostima ... Svi zadaci od 2.8. pa do 3.5. (225 stranica ...)	200 kn	99 kn	130 kn
1.E	Koordinatni sustav u ravnini Svi zadaci od 4.1. pa do 4.4. (250 stranica ...)	200 kn	120 kn	140 kn
10.	Fizika-1-po Mikuličić,Varičak,Vernić (mehanika)	240 kn	99 kn	99 kn
9.	Testovi po Dakiću Mat-1- komplet rješenja - Zbirke zadataka s pismenih ispita – Mat-1	555 kn	315 kn	360 kn
9.A	ili samo I – polugodište (I - II grupa)	200 kn	111 kn	150 kn
9.B	II- polugodište (III, V, VI, VII grupa)	355 kn	199 kn	266 kn
11.	Matematika -2- - Dakić-Elezović			
11.A	ili samo I - polugodište (I- IV poglavlja)	450 kn	199 kn	250 kn
11.B	II - polugodište (V - VII poglavlja)	450 kn	229 kn	295 kn
	Matematika -2- Dakić-Elezović po dijelovima:			
11.C	KOMPLEKSNI BROJEVI	100 kn	60 kn	80 kn
11.D	KVADRATNA JEDNADŽBA	120 kn	80 kn	90 kn
11.E	POLINOMI 2. STUPNJA	120 kn	80 kn	90 kn
11.F	TRIGONOMETRIJA pravokutnog trokuta	140 kn	80 kn	90 kn
12.	Fizika-2- po Mikuličić,Varičak,Vernić (toplina i elektricitet)	240 kn	140 kn	155 kn
13.	Testovi po Dakiću Mat-2- (I-VI grupa)	540 kn	310 kn	351 kn

	rješenja - Zbirke zadataka s pismenih ispita – Mat-2			
13.A	ili samo I – polugodište (I-IV grupa)	340 kn	199 kn	238 kn
13.B	II – polugodište (V-VI grupa)	200 kn	150 kn	170 kn
	ili svako poglavlje – za sebe	155 kn	50 kn	79 kn
14.	Matematika-3 - Dakić-Elezović			
14.A	ili samo I - polugodište (trigonometrija)	410 kn	199 kn	270 kn
14.B	II -polugodište (vektori i analitička geometrija)	440 kn	239 kn	289 kn
	Matematika-3 Dakić-Elezović - po DIJELOVIMA			
	1. polugodište			
14.A-1	Trigonometrija 1. dio – svi zadaci od. 1.1.- 3.3.	210 kn	99 kn	140 kn
14.A-2	Trigonometrija 2. dio – svi zadaci od. 3.3.- 6.5.	210 kn	99 kn	140 kn
	2. polugodište			
14.C	VEKTORI – svi zadaci od. 7.1. – do 7.7.	120 kn	80 kn	90 kn

Kod narudžbe matematičkih zbirki riješenih zadataka bitno je naglasiti da li je riječ o gimnazijskom programu ili o tehničkim školama ili matematičkim gimnazijama i o godini izdanja

M.I.M.-SRAGA d.o.o.

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Br.	Naziv ZBIRKE	Puna cijena	02.01.17.- 11.02.17. 30-60%	12.02.17.- 21.03.17. 15-40%
19.	Fizika-4 po Mikuličić, Varičak, Vernić (atomska i nuklearna)	90 kn	60 kn	68 kn
20.	Matematika-4- po Dakić Elezović za gimnazije (novo izdanje)	775 kn	375 kn	503 kn
20.A	ili samo I – polugodište (brojevi, nizovi, funkcije)	475 kn	199 kn	299 kn
20.B	II – polugodište (derivacije, integrali i pr funkcija)	360 kn	199 kn	250 kn
	ili po cjelinama ili poglavljima:			
20.D	1. grupa – BROJEVI – detaljno riješeni svi zadaci 1.1.-1.7. skripta na 190 stranica A4-format	175 kn	99 kn	120 kn
20.E	2. grupa – NIZOVI – detaljno riješeni svi zadaci 2.1.-2.7. skripta na 180 stranica A4-format	180 kn	99 kn	130 kn
20.F	3. grupa – FUNKCIJE – detaljno riješeni svi zad. 3.1.-3.5. skripta na 180 stranica A4-format	180 kn	99 kn	130 kn
19.	Matematika-4- po Dakić Elezović tehničke i matematičke gimnazije	900 kn	399 kn	540 kn
19.A	ili samo I – polugodište (brojevi, nizovi, funkcije)	475 kn	199 kn	299 kn
19.B	II – polugodište (derivacije, integrali i primitivna funkcija, dodatak-kombinatorika i vjerojatnost)	460 kn	299 kn	322 kn
21.	Testovi po Dakiću Mat-4- (I-VI grupa) komplet Rješenja zbirke zadataka s pismenih ispita B. Dakić	510 kn	306 kn	332 kn
21.A	ili samo – I polugodište. (I,II,III poglavlje)	255 kn	149 kn	179 kn
21.B	ili samo – II polugodište. (IV,V,VI poglavlje)	255 kn	166 kn	179 kn

Na ovom letku su ZBIRKE podijeljene po POGLAVLJIMA
 isto onako kako vam dolaze u školskim zbirkama
 dakle rješenja sada možete kupovati i po poglavljima

Za sve ostalo što nije u ovom cjeniku a treba vam slobodno nas nazovete ili nas kontaktirajte mailom ...

MOLIMO VAS DA OBRATITE PAŽNJU NA VRIJEME U KOJEM SE DAJU NAJVEĆI POPUSTI DA KASNIJE NE BI BILO NESPORAZUMA U POGLEDU CIJENA tj. svoje narudžbe napravite na vrijeme... (zbirke naručite odmah po najvećem popustu, a isporučit ćemo vam ih kada vam to financijski odgovara...)

POŠTARINA – Poštarina uz pakete s otkupninom je 25 kn

Moguća je kupovina po poglavljima, moguće je plaćanje na rate ili po dogovoru (nazovite za detaljne informacije)

Za sve ostale knjige iz naše ponude koje nisu u ovom cjeniku nazovite

ZA SVE INFORMACIJE I NARUDŽBE MOŽETE SE OBRATITI SVAKI DAN OD 09⁰⁰ – 20⁰⁰ ili mailom od 0-do-24

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