

**** MLADEN SRAGA ****
2011.

UNIVERZALNA ZBIRKA
POTPUNO RIJEŠENIH ZADATAKA
PRIRUČNIK ZA SAMOSTALNO UČENJE

MATEMATIKA

1

SKUP REALNIH BROJEVA

ALGEBARSKI RAZLOMCI

M.I.M.-SRAGA
 \sqrt{a}

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BESPLATNA - WEB-VARIJANTA

Tisak:

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Dodatne upute i objašnjenja možete zatražiti i na mail: mim-sraga@zg.htnet.hr

Ovo je jako skraćena varijanta naše zbirke ... samo oglednih 40-ak zadataka

M.I.M.-SRAGA d.o.o. zadržava sva prava na reproduciranje , umnažanje , prodaju ove zbirke potpuno riješenih zadataka isključivo u okviru svog programa poduke i dopisne poduke.

Nikakva komercijalna upotreba ove zbirke nije dozvoljena bez pismene dozvole nakladnika !

Ovo nisu svi zadaci iz ove zbirke ,

Ovo je samo manji dio od oko 12% zadataka iz kompletne zbirke ...

I ovdje su postavljeni samo kao ogledni primjerci

Ali vam mogu poslužiti kao solidna vježba pred testove ili ispitivanja u školi ...

201.

$$16) \frac{a^2 + b^2}{a^4 - b^4} = \frac{a^2 + b^2}{(a^2 - b^2)(a^2 + b^2)} = \frac{1 \cdot \cancel{(a^2 + b^2)}}{(a^2 - b^2) \cancel{(a^2 + b^2)}} = \frac{1}{a^2 - b^2}$$

↓

$$a^4 - b^4 = \underbrace{(a^2)^2 - (b^2)^2}_{\text{razlika kvadrata!}} = (a^2 - b^2)(a^2 + b^2)$$

↑

$$17) \frac{a^2 - b^2}{a^4 - b^4} = \frac{a^2 - b^2}{(a^2 - b^2)(a^2 + b^2)} = \frac{1 \cdot \cancel{(a^2 - b^2)}}{\cancel{(a^2 - b^2)}(a^2 + b^2)} = \frac{1}{a^2 + b^2}$$

$$19) \frac{x - y}{(y - x)^2} = \frac{x - y}{(x - y)^2} = \frac{x - y}{(x - y) \cdot (x - y)} = \frac{1 \cdot \cancel{(x - y)}}{\cancel{(x - y)} \cdot (x - y)} = \frac{1}{x - y}$$

↓

↑

$$(a - b)^2 = (b - a)^2 \quad \text{sjetimo se ovog pravila (pogledaj u naše formule !!)}$$

$$\begin{aligned} 20) \frac{x^2 y - xy^2}{3x^2 y^2 - 3xy^3} &= \frac{x \cdot x \cdot y - x \cdot y \cdot y}{3 \cdot x \cdot x \cdot y^2 - 3 \cdot x \cdot y^2 \cdot y^1} = \\ &= \frac{\underline{x} \cdot \underline{x} \cdot \underline{y} - \underline{x} \cdot \underline{y} \cdot \underline{y}}{\underline{3} \cdot \underline{x} \cdot \underline{x} \cdot \underline{y}^2 - \underline{3} \cdot \underline{x} \cdot \underline{y}^2 \cdot \underline{y}^1} = \\ &= \frac{x \cdot y \cdot (x - y)}{3 \cdot x \cdot y^2 \cdot (x - y)} = \frac{1 \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{(x - y)}}{3 \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{(x - y)}} = \\ &= \frac{1}{3y} \end{aligned}$$

ili kraće :

$$\frac{x^2 y - xy^2}{3x^2 y^2 - 3xy^3} = \frac{x \cdot y \cdot (x - y)}{3 \cdot x \cdot y^2 \cdot (x - y)} = \frac{1 \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{(x - y)}}{3 \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{(x - y)}} = \frac{1}{3y}$$

202.

$$3) \frac{a+b}{a^3+b^3} = \frac{a+b}{(a+b)(a^2-ab+b^2)} = \frac{\cancel{a+b}}{\cancel{a+b}(a^2-ab+b^2)} = \frac{1}{a^2-ab+b^2}$$

↓

treba prepoznati formulu za zbroj kubova

$$4) \frac{x^2-xy}{y^2-xy} = \frac{x \cdot x - x \cdot y}{y \cdot y - x \cdot y} =$$

$$= \frac{x \cdot (x-y)}{y \cdot (y-x)} = \frac{x \cdot (-y+x)}{y \cdot (y-x)} = \frac{x \cdot (-1) \cdot (y-x)}{y \cdot (y-x)} = \frac{x \cdot (-1) \cdot \cancel{(y-x)}}{y \cdot \cancel{(y-x)}} = x \cdot (-1) = -x$$

$$6) \frac{(x-y)^2}{x^2-y^2} = \frac{(x-y)(x-y)}{(x-y)(x+y)} = \frac{\cancel{(x-y)}(x-y)}{\cancel{(x-y)}(x+y)} = \frac{x-y}{x+y}$$

$$9) \frac{x^2-y^2}{x^3+y^3} = \frac{(x-y)(x+y)}{(x+y)(x^2-xy+y^2)} = \frac{(x-y)\cancel{(x+y)}}{\cancel{(x+y)}(x^2-xy+y^2)} = \frac{x-y}{x^2-xy+y^2}$$

↓

zbroj kubova

203.

$$\begin{aligned}
6.) \quad \frac{x^6 - y^6}{x^2 - y^2} &= \frac{(x^3)^2 - (y^3)^2}{(x-y)(x+y)} = \frac{(x^3 - y^3)(x^3 + y^3)}{(x-y)(x+y)} = \\
&= \frac{(x-y)(x^2 + xy + y^2)(x+y)(x^2 - xy + y^2)}{(x-y)(x+y)} = \\
&= \frac{\cancel{(x-y)}(x^2 + xy + y^2)\cancel{(x+y)}(x^2 - xy + y^2)}{\cancel{(x-y)}\cancel{(x+y)}} = \\
&= (x^2 + xy + y^2)(x^2 - xy + y^2) = \\
&= (x^2 + y^2 + xy)(x^2 + y^2 - xy) = \\
&= (x^2 + y^2)^2 - (xy)^2 = \\
&= (x^2)^2 + 2 \cdot x^2 \cdot y^2 + (y^2)^2 - x^2 y^2 = \\
&= x^4 + 2x^2 y^2 + y^4 - x^2 y^2 = x^4 + 2x^2 y^2 - 1 \cdot x^2 y^2 + y^4 = x^4 + x^2 y^2 + y^4
\end{aligned}$$

$$\begin{aligned}
8.) \quad \frac{(x-y)^2 - 1}{x^2 - x - y^2 - y} &= \frac{(x-y)^2 - 1^2}{x^2 - y^2 - x - y} = \frac{(x-y-1)(x-y+1)}{(x-y) \cdot (x+y) - 1 \cdot (x+y)} = \frac{(x-y-1)(x-y+1)}{(x+y) \cdot (x-y-1)} = \\
&= \frac{\cancel{(x-y-1)}(x-y+1)}{(x+y) \cdot \cancel{(x-y-1)}} = \frac{x-y+1}{x+y}
\end{aligned}$$

uputa: $(x-y)^2 - 1 = (x-y)^2 - 1^2 = (\text{to je razlika kvadrata ...}) = (x-y-1)(x-y+1)$

204.

$$1) \frac{x^2 y}{x^2 - x^2 y} = \frac{x^2 \cdot y}{x^2 \cdot 1 - x^2 \cdot y} = \frac{x^2 \cdot y}{x^2 \cdot (1 - y)} = \frac{\cancel{x^2} \cdot y}{\cancel{x^2} \cdot (1 - y)} = \frac{y}{1 - y}$$

$$5) \frac{4x^2 y - 4xy^2}{2x^2 y - 2xy^2} = \frac{4 \cdot x \cdot x \cdot y - 4 \cdot x \cdot y \cdot y}{2 \cdot x \cdot x \cdot y - 2 \cdot x \cdot y \cdot y} = \frac{4 \cdot x \cdot y \cdot (x - y)}{2 \cdot x \cdot y \cdot (x - y)} = \frac{2 \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{(x - y)}}{\cancel{2} \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{(x - y)}} = \frac{2}{1} = 2$$

ili kraće:

$$\frac{4x^2 y - 4xy^2}{2x^2 y - 2xy^2} = \frac{4 \cdot x \cdot y \cdot (x - y)}{2 \cdot x \cdot y \cdot (x - y)} = \frac{2 \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{(x - y)}}{\cancel{2} \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{(x - y)}} = \frac{2}{1} = 2$$

205.

$$7) \frac{a^4 b - a^2 b^3}{a^5 b - ab^5} = \frac{a^2 \cdot a^2 \cdot b^1 - a^2 \cdot b^1 \cdot b^2}{a^1 \cdot a^4 \cdot b^1 - a^1 \cdot b^1 \cdot b^4} =$$

$$= \frac{a^2 \cdot b^1 \cdot (a^2 - b^2)}{a^1 \cdot b^1 \cdot (a^4 - b^4)} = \frac{a \cdot a \cdot b \cdot (a^2 - b^2)}{a \cdot b \cdot (a^2 - b^2)(a^2 + b^2)} = \frac{\cancel{a} \cdot a \cdot \cancel{b} \cdot \cancel{(a^2 - b^2)}}{\cancel{a} \cdot \cancel{b} \cdot \cancel{(a^2 - b^2)} \cdot (a^2 + b^2)} = \frac{a}{a^2 + b^2}$$

dodatna uputa :

$$\frac{a^4 b - a^2 b^3}{a^5 b - ab^5} = \frac{a^{2+2} \cdot b^1 - a^2 \cdot b^{1+1}}{a^{1+4} \cdot b^1 - a \cdot b^{1+4}} =$$

$$= \frac{a^2 \cdot a^2 \cdot b^1 - a^2 \cdot b^1 \cdot b^2}{a^1 \cdot a^4 \cdot b^1 - a^1 \cdot b^1 \cdot b^4} = \quad \text{u brojniku i nazivniku podvučemo zajedničke faktore}$$

$$= \frac{a^2 \cdot b^1 \cdot (a^2 - b^2)}{a^1 \cdot b^1 \cdot (a^4 - b^4)} = \quad \text{izlučimo z.f. i u brojniku i u nazivniku}$$

$$= \frac{a \cdot a \cdot b \cdot (a^2 - b^2)}{a \cdot b \cdot (a^2 - b^2)(a^2 + b^2)} = \quad \text{u nazivniku treba prepoznati razliku kvadrata}$$

$$= \frac{\cancel{a} \cdot a \cdot \cancel{b} \cdot \cancel{(a^2 - b^2)}}{\cancel{a} \cdot \cancel{b} \cdot \cancel{(a^2 - b^2)} \cdot (a^2 + b^2)} = \quad \text{kratimo}$$

$$= \frac{a}{a^2 + b^2}$$

206.

$$\begin{aligned}
 10) \quad \frac{(x-3)^2}{(3-x)^3} &= \frac{(x-3)(x-3)}{(3-x)(3-x)(3-x)} = \frac{(x-3)(x-3)}{(-x+3)(-x+3)(3-x)} = \frac{(x-3)(x-3)}{-1 \cdot (x-3) \cdot (-1)(x-3) \cdot (3-x)} = \\
 &= \frac{(x-3)(x-3)}{(x-3)(x-3)(3-x)} = \frac{1 \cdot \cancel{(x-3)} \cdot \cancel{(x-3)}}{\cancel{(x-3)} \cdot \cancel{(x-3)} \cdot (3-x)} = \frac{1}{3-x}
 \end{aligned}$$

$$11) \quad \frac{x^3 + y^3}{(x+y)^3} = ?$$

$$\begin{aligned}
 12) \quad \frac{x^3 - y^3}{(x-y)^3} &= \frac{(x-y)(x^2 + xy + y^2)}{(x-y)^{1+2}} = \\
 &= \frac{(x-y)(x^2 + xy + y^2)}{(x-y)^1 \cdot (x-y)^2} = \frac{\cancel{(x-y)}(x^2 + xy + y^2)}{\cancel{(x-y)} \cdot (x-y)^2} = \frac{x^2 + xy + y^2}{(x-y)^2}
 \end{aligned}$$

208.

$$9) \frac{x^2 - y^2}{x^4 - y^4} = \frac{x^2 - y^2}{(x^2)^2 - (y^2)^2} = \frac{x^2 - y^2}{(x^2 - y^2)(x^2 + y^2)} = \frac{\cancel{(x^2 - y^2)}}{\cancel{(x^2 - y^2)}(x^2 + y^2)} = \frac{1}{x^2 + y^2}$$

$$12) \frac{1 - x^2}{x^2 - 2x + 1} = \frac{1^2 - x^2}{x^2 - 2x + 1^2} =$$

$$= \frac{(1-x)(1+x)}{(x-1)^2} = \frac{(1-x)(x+1)}{(1-x)^2} = \frac{(1-x)(x+1)}{(1-x)(1-x)} = \frac{\cancel{(1-x)}(x+1)}{\cancel{(1-x)}(1-x)} = \frac{x+1}{1-x}$$

$$\begin{array}{ccc} \downarrow & & \uparrow \\ \text{pravilo kaže: } (a-b)^2 & = & (b-a)^2 \end{array}$$

$$15) \frac{x^2 - x}{(1-x)^2} = \frac{x \cdot x - 1 \cdot x}{(x-1)^2} = \frac{x \cdot (x-1)}{(x-1)(x-1)} = \frac{x \cdot \cancel{(x-1)}}{(x-1) \cancel{(x-1)}} = \frac{x}{x-1}$$

$$\begin{array}{ccc} \downarrow & & \uparrow \\ \text{pravilo: } (a-b)^2 & = & (b-a)^2 \end{array}$$

209.

$$\begin{aligned}
 8) \quad \frac{x^3y + 2x^2y + xy}{x^3y - xy} &= \frac{x^1 \cdot x^2 \cdot y + 2 \cdot x^1 \cdot x^1 \cdot y + x^1 \cdot y \cdot 1}{x^1 \cdot x^2 \cdot y - x^1 \cdot y \cdot 1} = \\
 &= \frac{x^1 \cdot y \cdot (x^2 + 2 \cdot x + 1)}{x^1 \cdot y \cdot (x^2 - 1)} = \frac{\cancel{x} \cdot \cancel{y} \cdot (x+1)^2}{\cancel{x} \cdot \cancel{y} \cdot (x^2 - 1^2)} = \frac{(x+1)(x+1)}{(x-1)(x+1)} = \frac{(x+1)\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} = \frac{x+1}{x-1}
 \end{aligned}$$

210.

$$\begin{aligned}
 1) \quad \frac{(x-3)^2 + 12x}{x^2 - 9} &= \frac{x^2 - 2 \cdot x \cdot 3 + 3^2 + 12x}{x^2 - 3^2} = \quad \rightarrow \text{ pogledati u 57. zadatak kako se to radi ...} \\
 &= \frac{x^2 - 6x + 12x + 3^2}{(x-3)(x+3)} = \\
 &= \frac{x^2 + 6x + 3^2}{(x-3)(x+3)} = \frac{(x+3)^2}{(x-3)(x+3)} = \frac{(x+3)\cancel{(x+3)}}{(x-3)\cancel{(x+3)}} = \frac{x+3}{x-3}
 \end{aligned}$$

$$\begin{aligned}
 4) \quad \frac{(x+2)^2 - 8x}{x^2 - 5x - 6} &= \frac{x^2 + 4x + 4 - 8x}{x^2 - 3x - 2x - 6} = \frac{x^2 + 4x - 8x + 4}{x \cdot x - 3 \cdot x - 2 \cdot x - 2 \cdot 3} = \\
 &= \frac{x^2 - 4x + 4}{x \cdot (x-3) - 2 \cdot (x-3)} = \frac{(x-2)^2}{(x-3)(x-2)} = \frac{(x-2)\cancel{(x-2)}}{(x-3)\cancel{(x-2)}} = \frac{x-2}{x-3}
 \end{aligned}$$

uputa uz ovaj zadatak :

$x^2 - 5x - 6 \rightarrow$ je kvadratni trinom ...

tehnika kako se taj izraz rješava na faktore objašnjena je u 66. zadatku ove zbirke !

211.

$$\begin{aligned}
 9) \quad \frac{x^2 + y^2 - z^2 + 2xy}{x^2 + xz + xy - y - x - z} &= \frac{x^2 + 2xy + y^2 - z^2}{x \cdot (x + z + y) - (y + x + z)} = \frac{(x + y)^2 - z^2}{x \cdot (x + y + z) - 1 \cdot (y + x + z)} = \\
 &= \frac{(x + y - z)(x + y + z)}{(x + y + z) \cdot (x - 1)} = \frac{(x + y - z) \cancel{(x + y + z)}}{\cancel{(x + y + z)} \cdot (x - 1)} = \frac{x + y - z}{x - 1}
 \end{aligned}$$

212.

$$\begin{aligned}
 1) \quad \frac{x^8 - y^8}{x^3 + xy^2 - x^2y - y^3} &= \frac{(x^4)^2 - (y^4)^2}{x^1 \cdot x^2 + x \cdot y^2 - y \cdot x^2 - y^1 \cdot y^2} = \\
 &= \frac{(x^4 - y^4)(x^4 + y^4)}{x \cdot (x^2 + y^2) - y \cdot (x^2 + y^2)} = \\
 &= \frac{[(x^2)^2 - (y^2)^2](x^4 + y^4)}{(x^2 + y^2)(x - y)} = \\
 &= \frac{(x^2 - y^2)(x^2 + y^2)(x^4 + y^4)}{(x^2 + y^2)(x - y)} = \frac{(x^2 - y^2) \cancel{(x^2 + y^2)} (x^4 + y^4)}{\cancel{(x^2 + y^2)} (x - y)} = \\
 &= \frac{(x^2 - y^2)(x^4 + y^4)}{(x - y)} = \frac{\cancel{(x - y)} (x + y)(x^4 + y^4)}{\cancel{(x - y)}} = \\
 &= (x + y)(x^4 + y^4)
 \end{aligned}$$

213.

U ovom zadatku se ponovo susrećemo sa KVADRATNIM TRINOMOM

u zadatku br. 66. smo dali detaljnu uputu kako se kvadratni trinom rastavlja na faktore

$$1) \frac{x^2 - 4}{x^2 - x - 2} = \frac{x^2 - 2^2}{x^2 - 2x + 1x - 2} =$$

$$= \frac{(x-2)(x+2)}{x \cdot (x-2) + 1 \cdot (x-2)} = \frac{(x-2)(x+2)}{(x-2)(x+1)} = \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x+1)} = \frac{x+2}{x+1}$$

⇕

uputa kako rastaviti

kvadratni trinom iz brojnika:

$$x^2 - x - 2 = \left. \begin{array}{l} a = 1 \\ b = -1 \\ c = -2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} m + n = b \\ m \cdot n = a \cdot c \end{array} \right\} \Rightarrow \left. \begin{array}{l} m + n = -1 \\ m \cdot n = 1 \cdot (-2) \end{array} \right\} \Rightarrow \left. \begin{array}{l} m + n = -1 \\ m \cdot n = -2 \end{array} \right\} \Rightarrow m = -2, n = 1$$

$$m = -2, n = 1$$

$$x^2 - x - 2 = x^2 - 2x + 1x - 2 =$$

$$= x(x-2) - 1(x-2) =$$

$$= x(x-2) - 1(x-2) =$$

$$= (x-2)(x-1)$$

$$2) \frac{4x^2 - 4x + 1}{2x^2 - 5x + 2} = \frac{2^2 \cdot x^2 - 2 \cdot 2x \cdot 1 + 1^2}{2x^2 - 4x - 1x + 2} = \frac{(2x)^2 - 2 \cdot 2x \cdot 1 + 1^2}{2 \cdot x \cdot x - 2 \cdot 2 \cdot x - 1 \cdot x + 2} = \frac{(2x-1)^2}{2x \cdot (x-2) - 1 \cdot (x-2)} =$$

$$= \frac{(2x-1)(2x-1)}{(x-2)(2x-1)} = \frac{(2x-1)\cancel{(2x-1)}}{(x-2)\cancel{(2x-1)}} = \frac{2x-1}{x-2}$$

⇕

uputa kako rastaviti

kvadratni trinom iz brojnika:

$$2x^2 - 5x + 2 = \left. \begin{array}{l} a = 2 \\ b = -5 \\ c = 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} m + n = b \\ m \cdot n = a \cdot c \end{array} \right\} \Rightarrow \left. \begin{array}{l} m + n = -5 \\ m \cdot n = 2 \cdot 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} m + n = -5 \\ m \cdot n = 4 \end{array} \right\} \Rightarrow m = -4, n = -1$$

$$m = -4, n = -1$$

$$2x^2 - 5x + 2 = 2x^2 - 4x - 1x + 2 =$$

$$= 2x(x-2) - 1(x-2) =$$

$$= 2x(x-2) - 1(x-2) =$$

$$= (x-2)(2x-1)$$

214.

$$\begin{aligned}
4) \quad \frac{x^3 - x^2y + xy^2 - x^3}{x^3 + x^2y + xy^2 + y^3} &= \frac{x^2 \cdot x^1 - x^2 \cdot y + x^1 \cdot y^2 - x^1 \cdot x^2}{x^2 \cdot x^1 + x^2 \cdot y + x \cdot y^2 + y^2 \cdot y^1} = \\
&= \frac{x^2 \cdot (x - y) + x \cdot (y^2 - x^2)}{x^2 \cdot (x + y) + y^2 \cdot (x + y)} = \\
&= \frac{x \cdot x \cdot (x - y) + x \cdot (y - x)(y + x)}{(x + y)(x^2 + y^2)} = \frac{x \cdot x \cdot (x - y) - x \cdot (x - y)(y + x)}{(x + y)(x^2 + y^2)} = \\
&= \frac{x \cdot (x - y)[x - (y + x)]}{(x + y)(x^2 + y^2)} = \\
&= \frac{x \cdot (x - y)(x - y - x)}{(x + y)(x^2 + y^2)} = \frac{x \cdot (x - y)(-y)}{(x + y)(x^2 + y^2)} = \\
&= \frac{-xy(x - y)}{(x + y)(x^2 + y^2)} = \frac{-xy \cdot (-1) \cdot (-x + y)}{(x + y)(x^2 + y^2)} = \frac{+xy \cdot (y - x)}{(x + y)(x^2 + y^2)} = \\
&= \frac{xy(y - x)}{(x + y)(x^2 + y^2)}
\end{aligned}$$

215.

$$1) \frac{a^x + a^{x+1}}{a^x - a^{x+1}} = \frac{a^x \cdot 1 + a^x \cdot a^1}{a^x \cdot 1 - a^x \cdot a^1} = \frac{a^x \cdot (1+a)}{a^x \cdot (1-a)} = \frac{\cancel{a^x} \cdot (1+a)}{\cancel{a^x} \cdot (1-a)} = \frac{1+a}{1-a}$$

$$2) \frac{a^x + a^{x+1}}{a^x - a^{x+1}} = \frac{a^x \cdot 1 + a^x \cdot a^1}{a^x \cdot 1 - a^x \cdot a^1} = \frac{a^x \cdot (1+a)}{a^x \cdot (1-a)} = \frac{1+a}{1-a}$$

$$3) \frac{a^{x+2} - a^x}{a^{x+1} + a^x} = \frac{a^x \cdot a^2 - 1 \cdot a^x}{a^x \cdot a^1 + 1 \cdot a^x} =$$

$$= \frac{a^x \cdot (a^2 - 1)}{a^x \cdot (a+1)} = \frac{a^x \cdot (a-1)(a+1)}{a^x \cdot (a+1)} = \frac{\cancel{a^x} \cdot (a-1) \cancel{(a+1)}}{\cancel{a^x} \cdot \cancel{(a+1)}} = \frac{a-1}{1} = a-1$$

$$4) \frac{a^{x+4} - a^x}{a^{x+3} - a^{x+2} + a^{x+1} - a^x} = \frac{a^x \cdot a^4 - a^x \cdot 1}{a^x \cdot a^3 - a^x \cdot a^2 + a^x \cdot a^1 - a^x \cdot 1} = \frac{a^x \cdot (a^4 - 1)}{a^x \cdot (a^3 - a^2 + a - 1)} =$$

$$= \frac{\cancel{a^x} \cdot (a^4 - 1)}{\cancel{a^x} \cdot (a^3 - a^2 + a - 1)} =$$

$$= \frac{a^4 - 1}{a^3 - a^2 + a - 1} = \frac{(a^2)^2 - 1^2}{a^2 \cdot a^1 - a^2 \cdot 1 + 1 \cdot (a - 1)} =$$

$$= \frac{(a^2 - 1)(a^2 + 1)}{a^2 \cdot (a - 1) + 1 \cdot (a - 1)} =$$

$$= \frac{(a-1)(a+1)(a^2+1)}{(a-1)(a^2+1)} = \frac{\cancel{(a-1)}(a+1)\cancel{(a^2+1)}}{\cancel{(a-1)}\cancel{(a^2+1)}} =$$

$$= \frac{a+1}{1} = a+1$$

221.

$$\begin{aligned} 4) \quad \frac{x-1}{xy^2} + \frac{1-y}{x^2y} &= \frac{x-1}{x \cdot y \cdot y} + \frac{1-y}{x \cdot x \cdot y} = \frac{x \cdot (x-1) + y \cdot (1-y)}{x \cdot x \cdot y \cdot y} = \frac{x^2 - x + y - y^2}{x^2y^2} = \\ &= \frac{x^2 - y^2 - x + y}{x^2y^2} = \frac{(x-y)(x+y) - 1 \cdot (x-y)}{x^2y^2} = \frac{(x-y)(x+y-1)}{x^2y^2} \end{aligned}$$

5

$$\begin{aligned} 7) \quad \frac{2}{x-3} - \frac{4}{x+3} + \frac{3x}{x^2-9} &= \frac{2}{x-3} - \frac{4}{x+3} + \frac{3x}{x^2-9} = \frac{2}{x-3} - \frac{4}{x+3} + \frac{3x}{(x-3)(x+3)} = \\ &= \frac{2(x+3) - 4(x-3) + 3x}{(x-3)(x+3)} = \frac{2x+6-4x+12+3x}{(x-3)(x+3)} = \\ &= \frac{2x-4x+3x+6+12}{x^2-3^2} = \frac{x+18}{x^2-9} \end{aligned}$$

222.

$$\begin{aligned}
 2) \quad \frac{5x^2y^4z^4}{6a^3b^4c^5} \cdot \frac{18a^5b^4c}{25xy^2z^4} &= \frac{5 \cdot x \cdot x \cdot y^2 \cdot y^2 \cdot z^4}{6 \cdot a^3 \cdot b^4 \cdot c^1 \cdot c^4} \cdot \frac{6 \cdot 3 \cdot a^3 \cdot a^2 \cdot b^4 \cdot c}{5 \cdot 5 \cdot x \cdot y^2 \cdot z^4} = \\
 &= \frac{\cancel{6} \cdot \cancel{x} \cdot x \cdot \cancel{y^2} \cdot y^2 \cdot \cancel{z^4}}{\cancel{6} \cdot \cancel{a^3} \cdot \cancel{b^4} \cdot \cancel{c^1} \cdot c^4} \cdot \frac{\cancel{6} \cdot 3 \cdot \cancel{a^3} \cdot a^2 \cdot \cancel{b^4} \cdot \cancel{c}}{\cancel{6} \cdot 5 \cdot \cancel{x} \cdot y^2 \cdot \cancel{z^4}} = \\
 &= \frac{xy^2 \cdot 3 \cdot a^2}{c^4 \cdot 5} = \frac{3xy^2a^2}{5c^4}
 \end{aligned}$$

$$\begin{aligned}
 5) \quad \frac{x^3+y^3}{x-y} \cdot \frac{x^3-y^3}{x^2-xy+y^2} &= \frac{(x+y)(x^2-xy+y^2)}{(x-y)} \cdot \frac{(x-y)(x^2+xy+y^2)}{x^2-xy+y^2} = \\
 &= \frac{(x+y) \cancel{(x^2-xy+y^2)}}{\cancel{(x-y)}} \cdot \frac{\cancel{(x-y)} (x^2+xy+y^2)}{\cancel{(x^2-xy+y^2)}} = (x+y)(x^2+xy+y^2)
 \end{aligned}$$

225.

$$\begin{aligned}
 4) \quad \frac{x^2 + 4x + 4}{x^2 - y^2} : \frac{x^2 - 4}{x^2 - 2xy + y^2} &= \frac{x^2 + 2 \cdot 2 \cdot x + 2^2}{(x-y)(x+y)} \cdot \frac{x^2 - 2xy + y^2}{x^2 - 2^2} = \frac{(x+2)^2}{(x-y)(x+y)} \cdot \frac{(x-y)^2}{(x-2)(x+2)} = \\
 &= \frac{(x+2)\cancel{(x+2)}}{\cancel{(x-y)}(x+y)} \cdot \frac{\cancel{(x-y)}(x-y)}{(x-2)\cancel{(x+2)}} = \frac{(x+2)(x-y)}{(x+y)(x-2)}
 \end{aligned}$$

$$\begin{aligned}
 5) \quad \frac{(x-1)^2 - y^2}{(x+1)^2 - y^2} : \frac{x^2 - x - xy}{x^2 + x - xy} &= \frac{(x-1-y)(x-1+y)}{(x+1-y)(x+1+y)} \cdot \frac{x^2 + x - xy}{x^2 - x - xy} = \\
 &= \frac{(x-1-y)(x-1+y)}{(x+1-y)(x+1+y)} \cdot \frac{x \cdot (x+1-y)}{x \cdot (x-1-y)} = \\
 &= \frac{\cancel{(x-1-y)}(x-1+y)}{\cancel{(x+1-y)}(x+1+y)} \cdot \frac{\cancel{(x+1-y)}}{\cancel{(x-1-y)}} = \\
 &= \frac{x-1+y}{x+1+y} = \frac{x+y-1}{x+y+1}
 \end{aligned}$$

$$\begin{aligned}
 8) \quad \left(\frac{x-y}{x+y} + \frac{x+y}{x+y} \right) : \frac{x^3 + xy^2}{x^3 - xy^2} &= \frac{(x-y)(x-y) + (x+y)(x+y)}{(x+y) \cdot (x-y)} \cdot \frac{x^3 - xy^2}{x^3 + xy^2} = \\
 &= \frac{(x-y)^2 + (x+y)^2}{x^2 - y^2} \cdot \frac{x(x^2 - y^2)}{x(x^2 + y^2)} = \\
 &= \frac{x^2 - 2xy + y^2 + x^2 + 2xy + y^2}{x^2 - y^2} \cdot \frac{(x^2 - y^2)}{(x^2 + y^2)} = \\
 &= \frac{2x^2 + 2y^2}{x^2 - y^2} \cdot \frac{x^2 - y^2}{x^2 + y^2} = \frac{2\cancel{(x^2 + y^2)}}{\cancel{(x^2 - y^2)}} \cdot \frac{\cancel{(x^2 - y^2)}}{\cancel{(x^2 + y^2)}} = 2
 \end{aligned}$$

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