

x4. Rastavi na faktore koristeći se formulom za razliku kvadrata

$$a^2 - b^2 = (a-b) \cdot (a+b)$$

1) $a^2b^2 - c^2$

2) $100x^4 - 49y^6$

3) $x^4 - y^4$

4) $a^4b^6 - c^8$

5) $x^6 - y^6$

6) $x^8 - y^8$

7) $x^4 - 81$

8) $81x^4 - 1$

9) $16 - x^4$

10) $25 - a^4$

11) $625x^4 - 1$

12) $x^4y^2 - 9$

13) $x^2y^6 - 16$

14) $(a+b)^2 - 9$

15) $a^2 - (b-c)^2$

16) $a^4 - (a^2 + b^3)^2$

17) $25 - (x-3)^2$

18) $36 - (5-y)^2$

19) $81x^2 - (4x-3y)^2$

20) $(x^2+9)^2 - 36x^2$

21) $16x^2 - (x^2+4)^2$

22) $(x^2+x)^2 - 16$

23) $25 - (a^4 - b^2)^2$

24) $49 - (a^4 - 7)^2$

25) $(x^2 + y^2)^2 - 4x^2y^2$

26) $(x^2 + 9y^2)^2 - 36x^2y^2$

27) $25x^2 - (4x-3)^2$

28) $(25x^2 + 9y^2)^2 - 900x^2y^2$

29) $(x^2 - 2)^2 - (x^2 + 3)^2$

30) $(x^2 + y^2)^2 - 9$

31) $49x^2 - (5y-7x)^2$

32) $25x^2 - (y-x)^2$

33) $16a^2 - 9(a-b)^2$

34) $49(x-y)^2 - 25(x+y)^2$

35) $9(x-y)^2 - 16(x+y)^2$

36) $x^2 - y^2 - y + x$

37) $(x^2 + y^2)^2 - 4x^2y^2$

38) $(x^4 + 4y^6)^2 - 16x^4y^6$

39) $(x+y)^2 - 9x^4y^2$

40) $(x^2 - 6x)^2 - 81$

41) $625 - (x^2 - 10x)^2$

42) $(a^2 + 3a)^2 - 4$

43) $(a-b)^3 - 9(a-b)$

44) $(x^2 - 25)^2 + 9(x^2 - 25)$

45) $x^2(x-1) - y^2(x-1)$

46) $25(x+y)^2 - 16(x-y)^2$

47) $49(x^2 + y^3)^3 - 4(x^2 + y^3)$

48) $(x-1)^3 - 9(x-1)$

49) $a^5 - a^4 - a + 1$

50) $x^2 - 4x + 4 - y^2$

51) $a^4 - 2a^2 + 1 - b^2$

52) $x^2 - xy + 3y - 9$

53) $x^2 - xy - 5y - 25$

54) $a^2 - 2ab + b^2 - c^2$

55) $9a^2 - 12ab + 4b^2 - c^2$

$$1) \quad a^2b^2 - c^2 = (ab)^2 - c^2 = (ab - c) \cdot (ab + c)$$

$$\quad \quad \quad \downarrow \quad \quad \quad \downarrow$$

$$a^2 - b^2 = (a - b) \cdot (a + b)$$

$$2) \quad 100x^4 - 49y^6 = 10^2 \cdot (x^2)^2 - 7^2 \cdot (y^3)^2 =$$

$$= (10x^2)^2 - (7y^3)^2 = \quad \quad \quad \text{dobili smo izraz: } a^2 - b^2 \text{ rastavimo ga na faktore}$$

$$= (10x^2 - 7y^3) \cdot (10x^2 + 7y^3)$$

$$3) \quad x^4 - y^4 = (x^2)^2 - (y^2)^2 =$$

$$= (x^2 - y^2) \cdot (x^2 + y^2) =$$

$$= (x - y) \cdot (x + y) \cdot (x^2 + y^2)$$

Dodatno objašnjenje za one koji još nisu shvatili:

$$x^4 - y^4 = \underbrace{(x^2)^2 - (y^2)^2}_{\text{ovo je } a^2 - b^2} =$$

$$= \underbrace{(x^2 - y^2) \cdot (x^2 + y^2)}_{\text{ovo je } a^2 - b^2} =$$

$$= (x - y) \cdot (x + y) \cdot (x^2 + y^2)$$

$$4) \quad a^4b^6 - c^8 = (a^2b^3)^2 - (c^4)^2 =$$

$$= (a^2b^3 - c^4) \cdot (a^2b^3 + c^4)$$

Dodatno objašnjenje

$$a^4b^6 - c^8 = a^{2 \cdot 2} \cdot b^{3 \cdot 2} - c^{4 \cdot 2} = \rightarrow c^8 = c^{4 \cdot 2} = (c^4)^2$$

$$= (a^2)^2 \cdot (b^3)^2 - (c^4)^2 =$$

$$= (a^2b^3)^2 - (c^4)^2 =$$

$$= (a^2b^3 - c^4) \cdot (a^2b^3 + c^4)$$

$$\begin{aligned}
 5) \quad x^6 - y^6 &= x^{3 \cdot 2} - y^{3 \cdot 2} = \\
 &= (x^3)^2 - (y^3)^2 = \\
 &= (x^3 - y^3) \cdot (x^3 + y^3) = \\
 &= (x - y) \cdot (x^2 + xy + y^2) \cdot (x + y) \cdot (x^2 - xy + y^2) = \\
 &= (x - y) \cdot (x + y) \cdot (x^2 + xy + y^2) \cdot (x^2 - xy + y^2)
 \end{aligned}$$

Dodatno objašnjenje

$$\begin{aligned}
 x^6 - y^6 &= x^{3 \cdot 2} - y^{3 \cdot 2} = \\
 &= (x^3)^2 - (y^3)^2 = \quad \rightarrow \quad \text{ovdje imamo razliku kvadrata: } a^2 - b^2 \dots \\
 &= (x^3 - y^3) \cdot (x^3 + y^3) = \quad \rightarrow \quad \text{ovdje imamo razliku i zbroj kubova: } a^3 - b^3 \text{ i } a^3 + b^3 \\
 &= (x - y) \cdot (x^2 + xy + y^2) \cdot (x + y) \cdot (x^2 - xy + y^2) = \\
 &= (x - y) \cdot (x + y) \cdot (x^2 + xy + y^2) \cdot (x^2 - xy + y^2)
 \end{aligned}$$

$$\begin{aligned}
 6) \quad x^8 - y^8 &= x^{4 \cdot 2} - y^{4 \cdot 2} = \\
 &= (x^4)^2 - (y^4)^2 = \\
 &= (x^4 - y^4) \cdot (x^4 + y^4) = \\
 &= [(x^2)^2 - (y^2)^2] \cdot (x^4 + y^4) = \\
 &= (x^2 - y^2) \cdot (x^2 + y^2) \cdot (x^4 + y^4) = \\
 &= (x - y) \cdot (x + y) \cdot (x^2 + y^2) \cdot (x^4 + y^4)
 \end{aligned}$$

Dodatno objašnjenje

$$\begin{aligned}
 x^8 - y^8 &= x^{4 \cdot 2} - y^{4 \cdot 2} = \\
 &= \underbrace{(x^4)^2 - (y^4)^2}_{a^2 - b^2} = \quad \rightarrow \quad \text{ovdje imamo razliku kvadrata: } a^2 - b^2 \\
 &= (x^4 - y^4) \cdot (x^4 + y^4) = \\
 &= \underbrace{[(x^2)^2 - (y^2)^2]}_{a^2 - b^2} \cdot (x^4 + y^4) = \quad \rightarrow \quad \text{ovdje imamo R.KV.: } a^2 - b^2 \\
 &= \underbrace{(x^2 - y^2)}_{a^2 - b^2} \cdot (x^2 + y^2) \cdot (x^4 + y^4) = \quad \rightarrow \quad \text{ovdje imamo: } a^2 - b^2 \\
 &= (x - y) \cdot (x + y) \cdot (x^2 + y^2) \cdot (x^4 + y^4)
 \end{aligned}$$

$$\begin{aligned}
 7) \quad x^4 - 81 &= (x^2)^2 - 9^2 = \\
 &= (x^2 - 9) \cdot (x^2 + 9) = \\
 &= (x^2 - 3^2) \cdot (x^2 + 9) = \\
 &= (x - 3) \cdot (x + 3) \cdot (x^2 + 9)
 \end{aligned}$$

$$\begin{aligned}
 8) \quad 81x^4 - 1 &= 9^2 \cdot (x^2)^2 - 1^2 = \\
 &= (9x^2)^2 - 1^2 = \\
 &= (9x^2 - 1) \cdot (9x^2 + 1) = \\
 &= (3^2 x^2 - 1^2) \cdot (9x^2 + 1) = \\
 &= [(3x)^2 - 1^2] \cdot (9x^2 + 1) = \\
 &= (3x - 1) \cdot (3x + 1) \cdot (9x^2 + 1)
 \end{aligned}$$

$$\begin{aligned}
 9) \quad 16 - x^4 &= 4^2 - (x^2)^2 = \\
 &= (4 - x^2) \cdot (4 + x^2) = \\
 &= (2^2 - x^2) \cdot (x^2 + 4) = \\
 &= (2 - x) \cdot (2 + x) \cdot (x^2 + 4) = (2 - x) \cdot (x + 2) \cdot (x^2 + 4)
 \end{aligned}$$

$$\begin{aligned}
 10) \quad 25 - a^4 &= 5^2 - (a^2)^2 = \\
 &= (5 - a^2) \cdot (5 + a^2)
 \end{aligned}$$

$$\begin{aligned}
 11) \quad 625x^4 - 1 &= 25^2 \cdot (x^2)^2 - 1^2 = \\
 &= (25x^2)^2 - 1^2 = \quad \rightarrow \quad \text{R.KV.} \\
 &= (25x^2 - 1) \cdot (25x^2 + 1) = \\
 &= (5^2 \cdot x^2 - 1^2) \cdot (25x^2 + 1) = \\
 &= [(5x)^2 - 1^2] \cdot (25x^2 + 1) = \quad \rightarrow \text{R.KV.} \\
 &= (5x - 1) \cdot (5x + 1) \cdot (25x^2 + 1)
 \end{aligned}$$

$$\begin{aligned}
 12) \quad x^4y^2 - 9 &= (x^2)^2 \cdot y^2 - 3^2 = \\
 &= (x^2y)^2 - 3^2 = \\
 &= (x^2y - 3) \cdot (x^2y + 3)
 \end{aligned}$$

$$\begin{aligned}
 13) \quad x^2y^6 - 16 &= x^2 \cdot (y^3)^2 - 4^2 = \\
 &= (xy^3)^2 - 4^2 = \\
 &= (xy^3 - 4) \cdot (xy^3 + 4)
 \end{aligned}$$

$$\begin{aligned}
 14) \quad (a+b)^2 - 9 &= (a+b)^2 - 3^2 = \\
 &= (a+b-3) \cdot (a+b+3)
 \end{aligned}$$

$$\begin{aligned}
 15) \quad a^2 - (b-c)^2 &= [a - (b-c)] \cdot [a + (b-c)] = \\
 &= (a-b+c) \cdot (a+b-c)
 \end{aligned}$$

$$\begin{aligned}
 16) \quad a^4 - (a^2 + b^3)^2 &= (a^2)^2 - (a^2 + b^3)^2 = \\
 &= [a^2 - (a^2 + b^3)] \cdot [a^2 + (a^2 + b^3)] = \\
 &= (a^2 - a^2 - b^3) \cdot (a^2 + a^2 + b^3) = \\
 &= -b^3 \cdot (2a^2 + b^3)
 \end{aligned}$$

$$\begin{aligned}
 17) \quad 25 - (x-3)^2 &= 5^2 - (x-3)^2 = \\
 &= [5 - (x-3)] \cdot [5 + (x-3)] = \\
 &= (5-x+3) \cdot (5+x-3) = \\
 &= (8-x) \cdot (x+2)
 \end{aligned}$$

$$\begin{aligned}
 18) \quad 36 - (5-y)^2 &= 6^2 - (5-y)^2 = \\
 &= [6 - (5-y)] \cdot [6 + (5-y)] = \\
 &= (6-5+y) \cdot (6+5-y) = \\
 &= (1+y) \cdot (11-y) = (y+1) \cdot (11-y)
 \end{aligned}$$

$$\begin{aligned}
 19) \quad 81x^2 - (4x-3y)^2 &= 9^2 \cdot x^2 - (4x-3y)^2 = \\
 &= (9x)^2 - (4x-3y)^2 = \\
 &= [9x - (4x-3y)] \cdot [9x + (4x-3y)] = \\
 &= (9x - 4x + 3y) \cdot (9x + 4x - 3y) = \\
 &= (5x + 3y) \cdot (13x - 3y)
 \end{aligned}$$

$$\begin{aligned}
 20) \quad (x^2 + 9)^2 - 36x^2 &= (x^2 + 9)^2 - 6^2 \cdot x^2 = \\
 &= (x^2 + 9)^2 - (6x)^2 = \\
 &= (x^2 + 9 - 6x) \cdot (x^2 + 9 + 6x) = \\
 &= (x^2 - 6x + 9) \cdot (x^2 + 6x + 9) = \\
 &= (x^2 - 2 \cdot x \cdot 3 + 3^2) \cdot (x^2 + 2 \cdot x \cdot 3 + 3^2) = \\
 &= (x-3)^2 \cdot (x+3)^2
 \end{aligned}$$

Dodatno objašnjenje

$$\begin{aligned}
 (x^2 + 9)^2 - 36x^2 &= (x^2 + 9)^2 - 6^2 \cdot x^2 = \\
 &= \underbrace{(x^2 + 9)^2 - (6x)^2}_{a^2 - b^2} = && \text{imamo R.KV.} \\
 &= (x^2 + 9 - 6x) \cdot (x^2 + 9 + 6x) = \\
 &= (x^2 - 6x + 9) \cdot (x^2 + 6x + 9) = && \text{treba prepoznati: } (a^2 \pm 2ab + b^2) = (a \pm b)^2 \\
 &= \underbrace{(x^2 - 2 \cdot x \cdot 3 + 3^2)}_{\text{kvadrat razlike}} \cdot \underbrace{(x^2 + 2 \cdot x \cdot 3 + 3^2)}_{\text{kvadrat zbroja}} = \\
 &= (x-3)^2 \cdot (x+3)^2
 \end{aligned}$$

$$\begin{aligned}
 21) \quad 16x^2 - (x^2 + 4)^2 &= 4^2 \cdot x^2 - (x^2 + 4)^2 = \\
 &= (4x)^2 - (x^2 + 4)^2 = \\
 &= [4x - (x^2 + 4)] \cdot [4x + (x^2 + 4)] = \\
 &= (4x - x^2 - 4) \cdot (4x + x^2 + 4) = \\
 &= (-x^2 + 4x - 4) \cdot (x^2 + 4x + 4) = \\
 &= -1 \cdot (x^2 - 4x + 4) \cdot (x^2 + 2 \cdot x \cdot 2 + 2^2) = \\
 &= -1 \cdot \underbrace{(x^2 - 2 \cdot x \cdot 2 + 2^2)}_{a^2 - 2ab + b^2} \cdot \underbrace{(x^2 + 2 \cdot x \cdot 2 + 2^2)}_{a^2 + 2ab + b^2} = -1 \cdot (x-2)^2 \cdot (x+2)^2
 \end{aligned}$$

$$\begin{aligned}
 22) \quad & (x^2 + x)^2 - 16 = (x^2 + x)^2 - 4^2 = \\
 & = (x^2 + x - 4) \cdot (x^2 + x + 4)
 \end{aligned}$$

$$\begin{aligned}
 23) \quad & 25 - (a^4 - b^2)^2 = 5^2 - (a^4 - b^2)^2 = \\
 & = [5 - (a^4 - b^2)] \cdot [5 + (a^4 - b^2)] = \\
 & = (5 - a^4 + b^2) \cdot (5 + a^4 - b^2) = \\
 & = (-a^4 + b^2 + 5) \cdot (a^4 - b^2 + 5)
 \end{aligned}$$

$$\begin{aligned}
 24) \quad & 49 - (a^4 - 7)^2 = 7^2 - (a^4 - 7)^2 = \\
 & = [7 - (a^4 - 7)] \cdot [7 + (a^4 - 7)] = \\
 & = (7 - a^4 + 7) \cdot (7 + a^4 - 7) = \\
 & = (7 + 7 - a^4) \cdot (7 - 7 + a^4) = \\
 & = (14 - a^4) \cdot a^4 \\
 & = a^4 \cdot (14 - a^4)
 \end{aligned}$$

$$\begin{aligned}
 25) \quad & (x^2 + y^2)^2 - 4x^2y^2 = (x^2 + y^2)^2 - 2^2 \cdot x^2 \cdot y^2 = \\
 & = (x^2 + y^2) - (2xy)^2 = \\
 & = (x^2 + y^2 - 2xy) \cdot (x^2 + y^2 + 2xy) = \\
 & = (x^2 - 2xy + y^2) \cdot (x^2 + 2xy + y^2) = \\
 & = (x - y)^2 \cdot (x + y)^2
 \end{aligned}$$

$$\begin{aligned}
 26) \quad & (x^2 + 9y^2)^2 - 36x^2y^2 = (x^2 + 9y^2)^2 - 6^2 \cdot x^2 \cdot y^2 = \\
 & = (x^2 + 9y^2)^2 - (6xy)^2 = \\
 & = (x^2 + 9y^2 - 6xy) \cdot (x^2 + 9y^2 + 6xy) = \\
 & = (x^2 - 6xy + 3^2 \cdot y^2) \cdot (x^2 + 6xy + 3^2 \cdot y^2) = \\
 & = [x^2 - 2 \cdot x \cdot 3y + (3y)^2] \cdot [x^2 + 2 \cdot x \cdot 3y + (3y)^2] = \\
 & = (x - 3y)^2 \cdot (x + 3y)^2
 \end{aligned}$$

$$\begin{aligned}
 27) \quad 25x^2 - (4x-3)^2 &= 5^2 \cdot x^2 - (4x-3)^2 = \\
 &= (5x)^2 - (4x-3)^2 = \\
 &= [5x - (4x-3)] \cdot [5x + (4x-3)] = \\
 &= (5x - 4x + 3) \cdot (5x + 4x - 3) = \\
 &= (x+3) \cdot (9x-3)
 \end{aligned}$$

$$\begin{aligned}
 28) \quad (25x^2 + 9y^2)^2 - 900x^2y^2 &= (25x^2 + 9y^2)^2 - 30^2 \cdot x^2 \cdot y^2 = \\
 &= (25x^2 + 9y^2)^2 - (30xy)^2 = \\
 &= (25x^2 + 9y^2 - 30xy) \cdot (25x^2 + 9y^2 + 30xy) = \\
 &= (5^2 \cdot x^2 - 30xy + 3^2 \cdot y^2) \cdot (5^2 \cdot x^2 + 30xy + 3^2 \cdot y^2) = \\
 &= [(5x)^2 - 2 \cdot 5x \cdot 3y + (3y)^2] \cdot [(5x)^2 + 2 \cdot 5x \cdot 3y + (3y)^2] = \\
 &= (5x - 3y)^2 \cdot (5x + 3y)^2
 \end{aligned}$$

$$\begin{aligned}
 29) \quad (x^2 - 2)^2 - (x^2 + 3)^2 &= [x^2 - 2 - (x^2 + 3)] \cdot [x^2 - 2 + (x^2 + 3)] = \\
 &= (x^2 - 2 - x^2 - 3) \cdot (x^2 - 2 + x^2 + 3) = \\
 &= (x^2 - x^2 - 2 - 3) \cdot (x^2 + x^2 + 3 - 2) = \\
 &= -5 \cdot (2x^2 + 1)
 \end{aligned}$$

$$\begin{aligned}
 30) \quad (x^2 + y^2)^2 - 9 &= (x^2 + y^2)^2 - 3^2 = \\
 &= (x^2 + y^2 - 3) \cdot (x^2 + y^2 + 3)
 \end{aligned}$$

$$\begin{aligned}
 31) \quad 49x^2 - (5y - 7x)^2 &= 7^2 \cdot x^2 - (5y - 7x)^2 = \\
 &= (7x)^2 - (5y - 7x)^2 = \\
 &= [7x - (5y - 7x)] \cdot [7x + (5y - 7x)] = \\
 &= (7x - 5y + 7x) \cdot (7x + 5y - 7x) = \\
 &= (7x + 7x - 5y) \cdot (7x - 7x + 5y) \\
 &= (14x - 5y) \cdot 5y = \\
 &= 5y \cdot (14x - 5y)
 \end{aligned}$$

$$\begin{aligned}
 32) \quad 25x^2 - (y-x)^2 &= 5^2 \cdot x^2 - (y-x)^2 = \\
 &= (5x)^2 - (y-x)^2 = \\
 &= [5x - (y-x)] \cdot [5x + (y-x)] = \\
 &= (5x - y + x) \cdot (5x + y - x) = \\
 &= (6x - y) \cdot (4x + y)
 \end{aligned}$$

$$\begin{aligned}
 33) \quad 16a^2 - 9(a-b)^2 &= 4^2 \cdot a^2 - 3^2 \cdot (a-b)^2 = \\
 &= (4a)^2 - [3 \cdot (a-b)]^2 = \\
 &= (4a)^2 - (3a-3b)^2 = \\
 &= [4a - (3a-3b)] \cdot [4a + (3a-3b)] = \\
 &= (4a - 3a + 3b) \cdot (4a + 3a - 3b) = \\
 &= (a + 3b) \cdot (7a - 3b)
 \end{aligned}$$

$$\begin{aligned}
 34) \quad 49(x-y)^2 - 25(x+y)^2 &= 7^2 \cdot (x-y)^2 - 5^2 \cdot (x+y)^2 = \\
 &= [7 \cdot (x-y)]^2 - [5 \cdot (x+y)]^2 = \\
 &= (7x - 7y)^2 - (5x + 5y)^2 = \\
 &= [7x - 7y - (5x + 5y)] \cdot [7x - 7y + (5x + 5y)] = \\
 &= (7x - 7y - 5x - 5y) \cdot (7x - 7y + 5x + 5y) = \\
 &= (7x - 5x - 7y - 5y) \cdot (7x + 5x + 5y - 7y) = \\
 &= (2x - 12y) \cdot (12x - 2y) = \quad \text{to možemo ostaviti i u ovom obliku} \\
 &= 2 \cdot (x - 6y) \cdot 2 \cdot (6x - y) = \\
 &= 4 \cdot (x - 6y) \cdot (6x - y)
 \end{aligned}$$

$$\begin{aligned}
 35) \quad 9(x-y)^2 - 16(x+y)^2 &= 3^2 \cdot (x-y)^2 - 4^2 \cdot (x+y)^2 = \\
 &= [3 \cdot (x-y)]^2 - [4 \cdot (x+y)]^2 = \\
 &= (3x - 3y)^2 - (4x + 4y)^2 = \\
 &= [(3x - 3y) - (4x + 4y)] \cdot [(3x - 3y) + (4x + 4y)]^2 = \\
 &= (3x - 3y - 4x - 4y) \cdot (3x - 3y + 4x + 4y) = \\
 &= (-x - 7y) \cdot (7x + y) = \\
 &= -1 \cdot (x + 7y) \cdot (7x + y)
 \end{aligned}$$

$$\begin{aligned}
 36) \quad x^2 - y^2 - y + x &= (x-y) \cdot (x+y) + x - y = \\
 &= (x-y) \cdot (x+y) + 1 \cdot (x-y) = \\
 &= (x-y) \cdot (x+y+1)
 \end{aligned}$$

uputa:

$$\begin{aligned}
 \underbrace{x^2 - y^2}_{a^2 - b^2} - y + x &= \underbrace{(x-y) \cdot (x+y)}_{=(a-b) \cdot (a+b)} + x - y = && \text{prepoznaj razliku kvadrata (R.KV.)} \\
 &= (x-y) \cdot (x+y) + 1 \cdot (x-y) = && \text{izlučimo zajednički faktor: } (x-1) \\
 &= (x-y) \cdot (x+y+1)
 \end{aligned}$$

$$\begin{aligned}
 37) \quad (x^2 + y^2)^2 - 4x^2 y^2 &= (x^2 + y^2)^2 - 2^2 \cdot x^2 \cdot y^2 = \\
 &= (x^2 + y^2)^2 - (2xy)^2 = \\
 &= (x^2 + y^2 - 2xy) \cdot (x^2 + y^2 + 2xy) = \\
 &= (x^2 - 2xy + y^2) \cdot (x^2 + 2xy + y^2) = \\
 &= (x-y)^2 \cdot (x+y)^2
 \end{aligned}$$

$$\begin{aligned}
 38) \quad (x^4 + 4y^6)^2 - 16x^4 y^6 &= (x^4 + 4y^6)^2 - 4^2 \cdot (x^2)^2 \cdot (y^3)^2 = \\
 &= (x^4 + 4y^6)^2 - (4x^2 y^3)^2 = \\
 &= (x^4 + 4y^6 - 4x^2 y^3) \cdot (x^4 + 4y^6 + 4x^2 y^3) = \\
 &= (x^4 - 4x^2 y^3 + 4y^6) \cdot (x^4 + 4x^2 y^3 + 4y^6) = \\
 &= [(x^2)^2 - 2 \cdot x^2 \cdot 2 \cdot y^3 + (2y^3)^2] \cdot [(x^2)^2 + 2 \cdot x^2 \cdot 2 \cdot y^3 + (2y^3)^2] = \\
 &= (x^2 - 2y^3)^2 \cdot (x^2 + 2y^3)^2
 \end{aligned}$$

$$\begin{aligned}
 39) \quad (x+y)^2 - 9x^4 y^2 &= (x+y)^2 - 3^2 \cdot (x^2)^2 \cdot y^2 = \\
 &= (x+y)^2 - (3x^2 y)^2 = \\
 &= (x+y - 3x^2 y) \cdot (x+y + 3x^2 y)
 \end{aligned}$$

$$\begin{aligned}
 40) \quad & (x^2 - 6x)^2 - 81 = (x^2 - 6x)^2 - 9^2 = \\
 & = (x^2 - 6x - 9) \cdot (x^2 - 6x + 9) = \\
 & = (x^2 - 6x - 9) \cdot (x^2 - 2 \cdot x^2 \cdot 3 + 3^2) = \\
 & = (x^2 - 6x - 9) \cdot (x - 3)^2
 \end{aligned}$$

$$\begin{aligned}
 41) \quad & 625 - (x^2 - 10x)^2 = 25^2 - (x^2 - 10x)^2 = \\
 & = [25 - (x^2 - 10x)] \cdot [25 + (x^2 - 10x)] = \\
 & = (25 - x^2 + 10x) \cdot (25 + x^2 - 10x) = \\
 & = (-x^2 + 10x + 25) \cdot (x^2 - 10x + 25) = \\
 & = (-x^2 + 10x + 25) \cdot (x^2 - 2 \cdot x \cdot 5 + 5^2) = \\
 & = (-x^2 + 10x + 25) \cdot (x - 5)^2
 \end{aligned}$$

$$\begin{aligned}
 42) \quad & (a^2 + 3a)^2 - 4 = (a^2 + 3a)^2 - 2^2 = \\
 & = (a^2 + 3a - 2) \cdot (a^2 + 3a + 2)
 \end{aligned}$$

$$\begin{aligned}
 43) \quad & (a - b)^3 - 9(a - b) = (a - b)^{1+2} - 9 \cdot (a - b) = \\
 & = (a - b)^1 \cdot (a - b)^2 - 9 \cdot (a - b)^1 = \\
 & = (a - b)^1 \cdot [(a - b)^2 - 9] = \\
 & = (a - b) \cdot [(a - b)^2 - 3^2] = \\
 & = (a - b) \cdot (a - b - 3) \cdot (a - b + 3)
 \end{aligned}$$

dodatna uputa:

$$\begin{aligned}
 (a - b)^3 &= (a - b)^{1+2} = (a - b)^1 \cdot (a - b)^2 \\
 (a - b)^3 - 9(a - b) &= (a - b)^{1+2} - 9 \cdot (a - b) = \\
 &= (a - b)^1 \cdot (a - b)^2 - 9 \cdot (a - b)^1 = \\
 &= (a - b)^1 \cdot [(a - b)^2 - 9] = \\
 &= (a - b) \cdot \underbrace{[(a - b)^2 - 3^2]}_{a^2 - b^2} = \\
 &= (a - b) \cdot (a - b - 3) \cdot (a - b + 3)
 \end{aligned}
 \quad \left. \begin{array}{l} \text{izlučimo z.f. } (a - b)^1 \\ \text{u uglatoj zagradi imamo R.K.} \end{array} \right\}$$

$$\begin{aligned}
 44) \quad & (x^2 - 25)^2 + 9(x^2 - 25) = (x^2 - 25) \cdot (x^2 - 25) + 9 \cdot (x^2 - 25) = \\
 & = (x^2 - 25) \cdot [(x^2 - 25) + 9] = \\
 & = (x^2 - 5^2) \cdot (x^2 - 25 + 9) = \\
 & = (x - 5) \cdot (x + 5) \cdot (x^2 - 16) = \\
 & = (x - 5) \cdot (x + 5) \cdot (x^2 - 4^2) = \\
 & = (x - 5) \cdot (x + 5) \cdot (x - 4) \cdot (x + 4)
 \end{aligned}$$

$$\begin{aligned}
 45) \quad & x^2(x - 1) - y^2(x - 1) = (x - 1) \cdot (x^2 - y^2) = \\
 & = (x - 1) \cdot (x - y) \cdot (x + y)
 \end{aligned}$$

$$\begin{aligned}
 46) \quad & 25(x + y)^2 - 16(x - y)^2 = 5^2 \cdot (x + y)^2 - 4^2 \cdot (x - y)^2 = \\
 & = [5 \cdot (x + y)]^2 - [4 \cdot (x - y)]^2 = \\
 & = (5x + 5y)^2 - (4x - 4y)^2 = \\
 & = [(5x + 5y) - (4x - 4y)] \cdot [(5x + 5y) + (4x - 4y)] = \\
 & = (5x + 5y - 4x + 4y) \cdot (5x + 5y + 4x - 4y) = \\
 & = (x + 9y) \cdot (9x + y)
 \end{aligned}$$

$$\begin{aligned}
 47) \quad & 49(x^2 + y^3)^3 - 4(x^2 + y^3) = 49(x^2 + y^3)^1 \cdot (x^2 + y^3)^2 - 4(x^2 + y^3)^1 = \\
 & = (x^2 + y^3) \cdot [49(x^2 + y^3)^2 - 4] = \\
 & = (x^2 + y^3) \cdot (7^2 \cdot (x^2 + y^3)^2 - 2^2) = \\
 & = (x^2 + y^3) \cdot [(7 \cdot (x^2 + y^3))^2 - 2^2] = \\
 & = (x^2 + y^3) \cdot [(7x^2 + 7y^3)^2 - 2^2] = \\
 & = (x^2 + y^3) \cdot (7x^2 + 7y^3 - 2) \cdot (7x^2 + 7y^3 + 2)
 \end{aligned}$$

$$\begin{aligned}
 48) \quad (x-1)^3 - 9(x-1) &= (x-1)^1 \cdot (x-1)^2 - 9 \cdot (x-1) = \\
 &= (x-1) \cdot [(x-1)^2 - 9] = \\
 &= (x-1) \cdot [(x-1)^2 - 3^2] = \\
 &= (x-1) \cdot (x-1-3) \cdot (x-1+3) = \\
 &= (x-1) \cdot (x-4) \cdot (x+2)
 \end{aligned}$$

$$\begin{aligned}
 49) \quad a^5 - a^4 - a + 1 &= a^{4+1} - a^4 - 1 \cdot (a-1) = \\
 &= a^4 \cdot a^1 - a^4 \cdot 1 - 1 \cdot (a-1) = \\
 &= a^4 \cdot (a-1) - 1 \cdot (a-1) = \\
 &= (a-1) \cdot (a^4 - 1) = \\
 &= (a-1) \cdot [(a^2)^2 - 1^2] = \\
 &= (a-1) \cdot (a^2 - 1) \cdot (a^2 + 1) = \\
 &= (a-1) \cdot (a-1) \cdot (a+1) \cdot (a^2 + 1)
 \end{aligned}$$

Dodatna uputa:

$$\begin{aligned}
 a^5 - a^4 - a + 1 &= a^{4+1} - a^4 - 1 \cdot (a-1) = &\rightarrow a^5 = a^{4+1} = a^4 \cdot a^1 \\
 &= \underbrace{a^4 \cdot a^1 - a^4 \cdot 1}_{\text{izlučimo z.f.}} - 1 \cdot (a-1) = &\rightarrow \text{u prva dva člana izlučimo z.f. } a^4 \\
 &= a^4 \cdot (a-1) - 1 \cdot (a-1) = &\rightarrow \text{izlučimo z.f. } (a-1) \\
 &= (a-1) \cdot \underbrace{(a^4 - 1)}_{a^2 - b^2} = &\rightarrow \text{treba prepoznati R.KV.} \\
 &= (a-1) \cdot [(a^2)^2 - 1^2] = \\
 &= (a-1) \cdot \underbrace{(a^2 - 1)}_{a^2 - b^2} \cdot (a^2 + 1) = &\rightarrow \text{srednja zagrada je opet R.KV.} \\
 &= (a-1) \cdot (a-1) \cdot (a+1) \cdot (a^2 + 1)
 \end{aligned}$$

$$\begin{aligned}
 50) \quad x^2 - 4x + 4 - y^2 &= x^2 - 2 \cdot x \cdot 2 + 2^2 - y^2 = \\
 &= (x-2)^2 - y^2 = \\
 &= (x-2-y) \cdot (x-2+y) = \\
 &= (x-y-2) \cdot (x+y-2)
 \end{aligned}$$

Dodatna uputa:

$$\underbrace{x^2 - 4x + 4}_{\substack{\text{ovo je} \\ \text{kvadrat razlike}}} - y^2 = \underbrace{x^2 - 2 \cdot x \cdot 2 + 2^2}_{a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2} - y^2 = \underbrace{(x-2)^2 - y^2}_{a^2 - b^2} = (x-2-y) \cdot (x-2+y)$$

$$\begin{aligned}
 51) \quad a^4 - 2a^2 + 1 - b^2 &= (a^2)^2 - 2 \cdot a \cdot 1 + 1^2 - b^2 = \\
 &= (a^2 - 1)^2 - b^2 = \\
 &= (a^2 - 1 - b) \cdot (a^2 - 1 + b) = \\
 &= (a^2 - b - 1) \cdot (a^2 + b - 1)
 \end{aligned}$$

$$\begin{aligned}
 52) \quad x^2 - xy + 3y - 9 &= x^2 - 9 - xy + 3y = \\
 &= x^2 - 3^2 - x \cdot y + 3 \cdot y = \\
 &= (x - 3) \cdot (x + 3) - y \cdot (x - 3) \\
 &= (x - 3) \cdot (x + 3 - y)
 \end{aligned}$$

$$\begin{aligned}
 53) \quad x^2 - xy - 5y - 25 &= x^2 - 25 - xy - 5y = \\
 &= x^2 - 5^2 - x \cdot y - 5 \cdot y = \\
 &= (x - 5) \cdot (x + 5) - y \cdot (x + 5) = \\
 &= (x + 5) \cdot (x - 5 - y)
 \end{aligned}$$

$$\begin{aligned}
 54) \quad a^2 - 2ab + b^2 - c^2 &= (a - b)^2 - c^2 = \\
 &= (a - b - c) \cdot (a - b + c)
 \end{aligned}$$

$$\begin{aligned}
 55) \quad 9a^2 - 12ab + 4b^2 - c^2 &= 3^2 \cdot a^2 - 2 \cdot 3 \cdot 2 \cdot a \cdot b + 2^2 \cdot b^2 - c^2 = \\
 &= (3a)^2 - 2 \cdot 3a \cdot 2b + (2b)^2 - c^2 = \\
 &= (3a - 2b)^2 - c^2 = \\
 &= (3a - 2b - c) \cdot (3a - 2b + c)
 \end{aligned}$$